STEREOMETRIE:

OR,

The ART of PRACTICAL GAUGING,

Shewing in *TWO PARTS*,

FIRST,

Divers facil and compendious ways for Gauging of TUNNS and *Brewers* VESSELS, of all *Forms* and *Figures*, either in whole, or gradualy, from Inch to Inch:

Whether the Tunn, or Vessels Bases above and below be { *Homogeneal*, or *Heterogeneal*. Paralliel and alike-situate, or not.

SECONDLY,

The Gauging of any *Wine*, *Brandy*, or *Oyl CASK*; be the same assum'd as *Sphæroidal*, *Parabolical*, *Conical*, or *Cylindrical*; either full, or partly empty, and at any *Position* of the *Cask*, or *Altitude* of contained *Liquor*: Performed either by brief *Calculation*, or *Instrumental Operation*.

TOGETHER WITH

A large TABLE of Area's of a Circles Segments, and other necessary TABLES, & their excellent Utilities and Emprovements; with a Copious and Methodical INDEX of the Whole; rendring the Work perspicuous and intelligible to Mean Capacities.

By *JOHN SMITH*, Philo-Accomptant.

Omne tulit punctum, qui miscuit utile dulci.

London, Printed by *William Godbid*, for *William Shrowsbury*, at the *Bible* in *Duck-Lane*. 1673.

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To the Right Worſhipful

Sir WILLIAM BUCKNALL,

KNIGHT & BARONET;

John & Danet Fourth,

ESQUIRES;

William & George Daſhwood,

ESQUIRES;

And the reſt of the FARMERS

of His Majeſties Duty of *Exciſe*:

AND UNTO

Robert Holden, Eſq;

John Smith,

To expreſs his humble Gratitude

And ſingular Obſervance,

DEDICATES

Theſe his LABOURS.

To the Right Worshipful

Sir WILLIAM BUCKNALL,

KNIGHT & BARONET;

John & Daniel Fourth,

ESQUIRES;

William & George Dalbywood,

ESQUIRES;

And the rest of the FARMERS

of His Majesties Duty of Excise;

AND UNTO

Robert Holden, Esq;

John Smith,

To express his humble Gratitude

And singular Obligations

DEDICATES

This his LAWFUL

TO THE READER.

Courteous Reader,

TO thy candid cenſure and favourable reception is here ſubmitted a ſmall Treatiſe of *Stereometrie*, or *Solid Geometrie*, I having in this ſcribling Age caſt this Mite (ſuch as it is) into the Publick Treaſury (*Scribimus enim Indocti, &c.*) In which Work, for many ingenious Perſons ſakes, who having a competent readineſs in the Rule of *Multiplication*, are yet unskilful in *Diviſion*, I have performed the Work thoroughout by *Multiplication*; and doubt not but it will be evident, that I balk not any material thing by ſilence, or cloud any thing by words, yet true it is, the Work hath none of thoſe Embelliſhments, which a Poliſh'd or Learned Pen might have adorned it with,

and ſome of the *Problems* (by reaſon of ſeveral diſcompoſures and obſtacles beſieging me) have not that Methodical Collocation, that I firſt intended: The which I have endeavoured to obviate and remove, by a copious and particular *Synopſis*, or *Index*, before the Book; which (exhibiting in few words and little room a ſummary Repreſentation of the whole Book) will by ocular Inſpection, on a ſudden direct thee unto the ſubject Matter of thy Deſire, in any part of the Work: And (having of late had too much leiſure time from other Employments) I have with great Induſtry and Care Calculated three Tables, *viz.* firſt, that long-look'd for Table of *Area's* of a *Circles Segments*; then, two Tables, call'd *Cyclometrical Tables*, the one, ſhewing the Area's of all Circles in Ale Gallons, and Decimilleſſimal Parts, from one Inch, to ſix Yards Diameter; the other, exhibiting all Circles Area's in Wine Gallons and Decimilleſſimal Parts, from one Inch unto five Feet Diameter; both Calculated to every Tenth Part and Quar-

Quarters of an Inch; laying down the Conſtruction, Riſe, and Fabrick, as well as the Utility and Emprovement of each Table in particular, as is very conſpicuous in *pag.* 236, *&c.* Concerning all which Tables, I may further confidently ſay, ſuch and ſo great hath been my own and the Printers vigilance and care, that I find no one numeral Miſtake in them, and am not in the leaſt conſcious to my ſelf of any remiſneſs in their Calculations; yet of ſome Miſtakes elſewhere in the Book you have Advertiſement, which I deſire (before reading the Book) to be corrected.

But if it be objected, That of divers Multiplicators, the Riſe and Fabrick is not given: To this I anſwer, I did indeed at firſt intend the inſerting of the way for conſtituting every particular Multiplicator, as I have of .0034+ and .0027851 in *pag.* 233. .0035461+ in *pag.* 21. and .0000985028+ in *pag.* 22. but finding many of them very much complicated, the denodation and unravelling them would coſt many words

and

*As appears by pag. 233, and 234.

and much *paper, and ſo not only render the Book more chargeable, but voluminous, and beyond the bulk of a pocketable *Vade-Mecum*, (Contraction having been all along deſigned.) Moreover, from the Riſe and Conſtitution of thoſe Multiplicators given, (as aforeſaid) may the Riſe of any other in the Book (*mutatis mutandis*) be with facility found out: To all which I may further add, the Multiplicators are truly Calculated and ſo Printed (the *Errata's* advertis'd of being excepted or corrected) as in *pag.* 198, 204. and in moſt Places of the Book, the Multiplicators appear true, * that is (being ſurd Numbers) *quàm proximè verùm.*

*Moſt of the Queſtiõs being wrought divers ways, *viz.* as well with, as without the Multiplicators.

Yet leſt I now expoſe this Treatiſe unto prejudice, give me leave to premiſe theſe few Advertiſements, *viz.* Firſt, it is requiſite, that he, who intends to reap any benefit by reading this Book, ſhould have a proper Genius, not only ready to conceive Mathematical Notions, but apt likewiſe to take a kind of pleaſure in them: For, *de quolibet ligno non fit Mercurius.* Again, it is expected he

he ſhould be aforehand furniſh'd with a competent knowledge of the Terms of Art in *Geometrie*, and have a Notion of Solid Bodies, as a Cube, Globe, Cone, Cylinder, Sphæroid, *&c.* and of ſuperficial Figures, as a Circle, an Ellipſis, a Triangle, a Parallelogram, a Square, *&c.* And laſtly, he ought in ſome meaſure to be acquainted with the Nature of Numbers, whole and broken, at the leaſt with *Numeration*, *Addition*, *Subduction*, and *Multiplication*, but more eſpecially with *Decimal Fractions, and the Nature of them, in ſome competent degree at leaſt.

Vide Hodder's *Arithm.*

For a Body of the Mathematicks was never intended, nor can be rationally expected in this ſmall Bulk; and therefore for Demonſtrations and larger Explanations of the Terms of Art, *&c.* (contain'd in this Treatiſe) I refer the Reader to the further Scrutiny of other Authors; yet if he have only a ſuperficial knowledge of, or be but in part acquainted with the Terms of Art aforeſaid, and a Decimal Fraction; he (being induſtrious) by diligent peruſal and

and ſerious obſervation of the Work, and comparing the *Problems* with their reſpective and proper *Schemes*, may better his knowledge, and much improve himſelf by this ſmall Work.

It is not the Intendment of this Epiſtolar Diſcourſe to perſwade my Reader of the uſefulneſs and utility, or rather neceſſity of the *Geometrical Science*, or *Art Metrical*: The Rational and Convincing Prefaces of many Learned Men, the heavy progreſs of former times much wanting it, with the general acceptance of, and great improvement by theſe Helps in later times, render ſuch Labours altogether uſeleſs and ineffectual; yea, through the want of skill in *Solid Geometrie*, much injuſtice is daily committed by our *Common Manufactures, both unto others and themſelves.

**As by ſome unskilful Gaugers, Carpenters, Timber-Merchants, &c.*

Thus hoping that my real intention to advance the Publick Good, will procure from the ingenuous Reader, a favourable Conſtruction of what he ſhall therein find not wilfully miſtaken; the which if granted, and that the Labour

bour (herein taken)do find good reception by the *Tyro* and young Practitioner, (for whoſe uſe it is principally intended,) I ſhall when opportunity preſents, be encourag'd further to explicate and refine this Work, to contract the ſame in ſome, and enlarge it in other places; and alſo to expoſe to publick view ſome other Pieces remaining by me in *Manuſcripts*, which I have not (though ſollicited to their Publication) hitherto thought worthy to ſee the light; one is a Treatiſe of *Compound Intereſt*; another, of above an hundred Extraordinary Caſes in *Plane Trigonometry*; another, of *Arithmetical Rarities*; and laſtly, my ſmall Piece of *Rhetorique*, new modell'd, and render'd more perſpicuous, intelligible, and delightful: Of each of the three foremoſt, doth after the Epiſtle follow a Taſte.

But I fear I have made my Gate too broad, in proportion unto the narrow Walls of the City, and will therefore detain thee no longer in the Porch, but invite thee into the Houſe, where, ſuch as I have, is ſet before thee; and

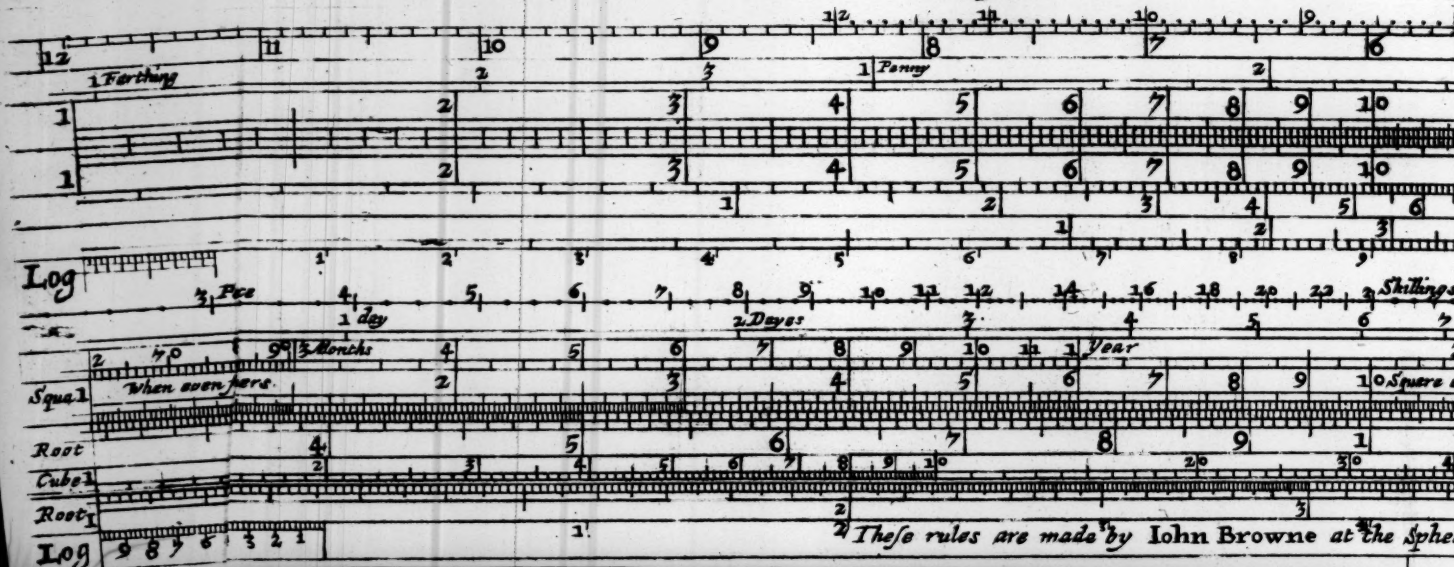
and if thou find not such chear and variety, as thou expectedst; yet grant thy Acceptance and friendly Resentment, since it is according to the ability of

From my Chamber, at the House of Captain Robert Holden, *Oct.* 19. 1672.

Thy real Well-wisher,

John Smith.

Hy-



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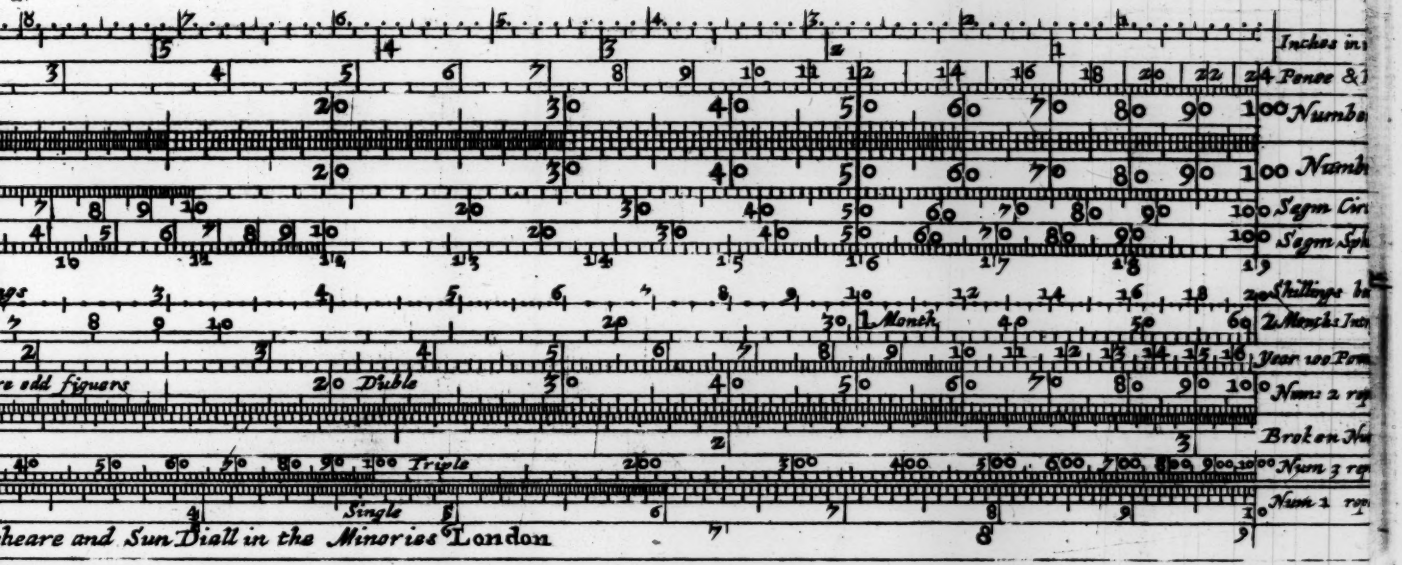
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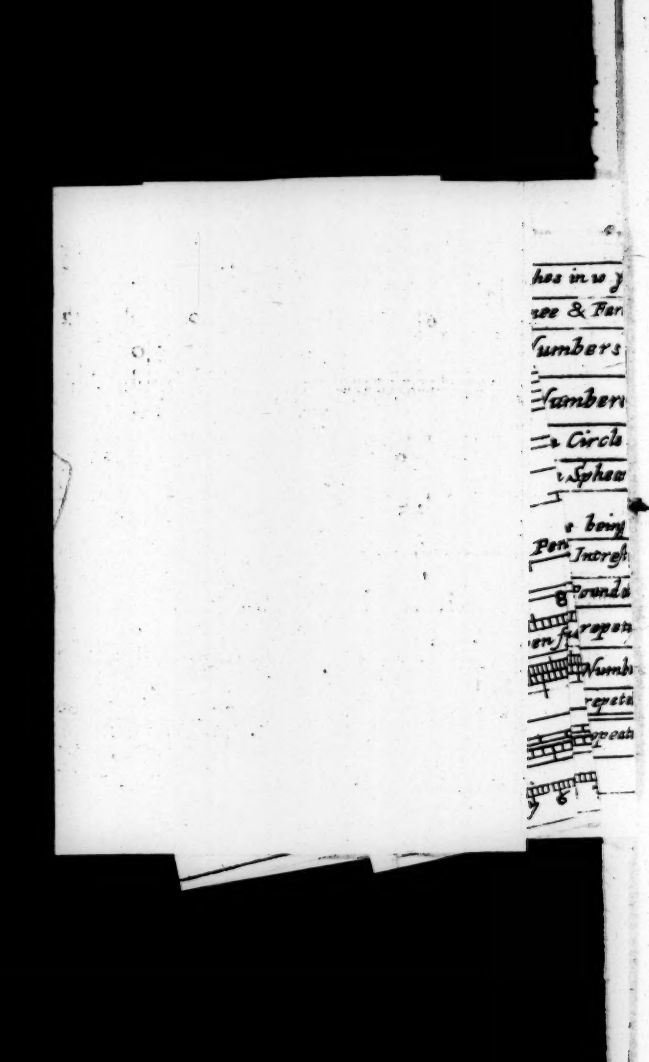


Fig. 23

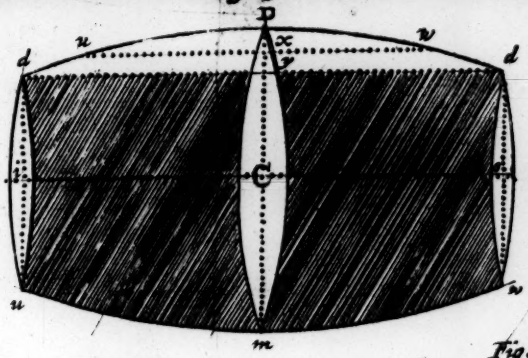


Fig. 24

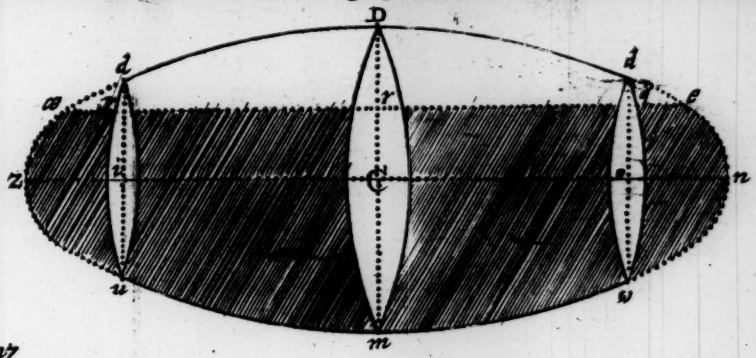


Fig. 25

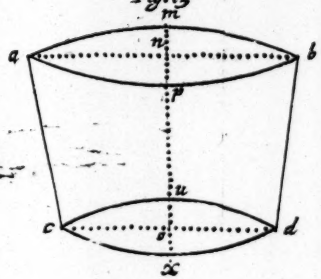


Fig. 27

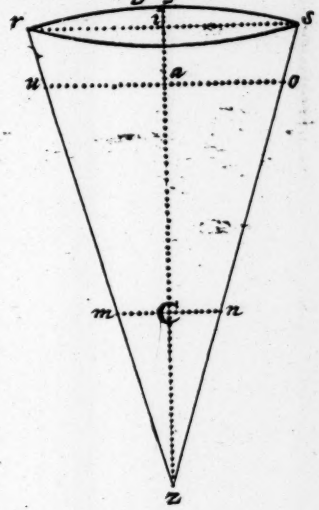
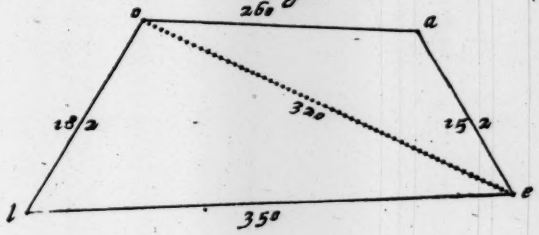


Fig. 26



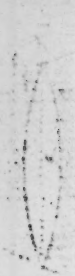


Fig. 1

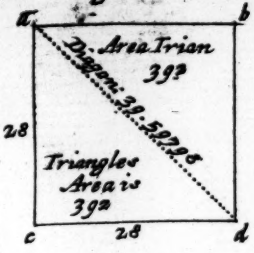


Fig. 2

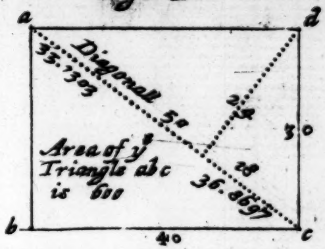


Fig. 3

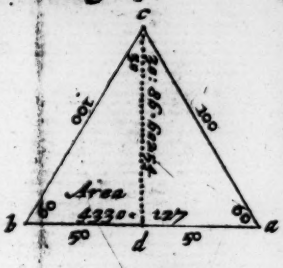


Fig. 4

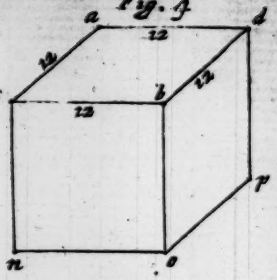


Fig. 5

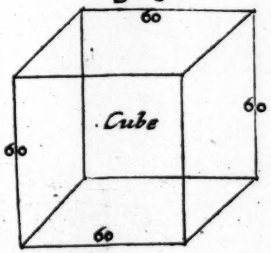
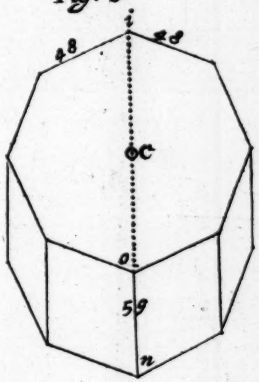


Fig. 6



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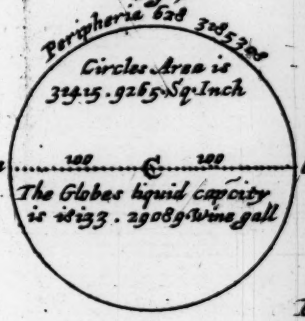


Fig. 8

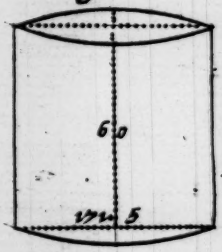


Fig. 9

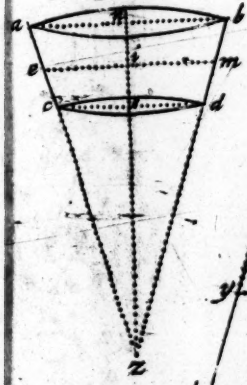


Fig. 10

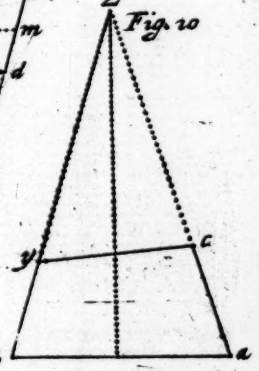


Fig. 11

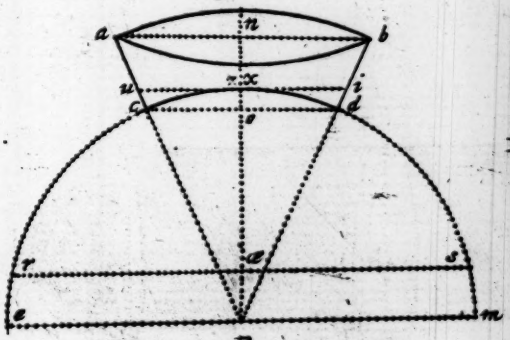
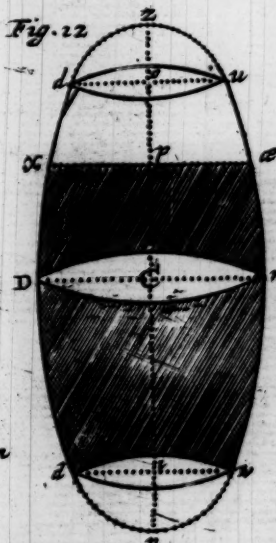


Fig. 12



A Taſte of ſome Manuſcripts, *lying by me, for the Entertainment of ſome ſpare Minutes and exerciſe of Art.*

THere is an Annuity, or Yearly Payment of an 100 *l.* for a term of 115 Years to be ſold: Whether is it better for the Purchaſer to buy the ſame at *6 l. per Cent. per Annum*, Simple Intereſt; or at *6 l. per Cent. per Annum*, Compound Intereſt; and by how much? What alſo is its Value at *8 l. per Cent. per Annum*, Compound Intereſt?

A takes up 300 *l.* of B, ſecuring a Payment of 5 *l.* quarterly unto B, until he be reimburſt his Principal, with Intereſt upon Intereſt, after the Rate of *6 l. per Cent.* How many full Quarters Rent muſt B receive, and what ought his laſt Receipt to be?

OF an Oblique-angular Plane Triangle, having the Vertical Angle = 116 deg. 25′ or 116.4166667+ deg. a Leg = 350, and the Altern Segment of the Baſe (made by the Perpendicular) = 410: To find the Quantity of the Baſe, or Perpendicular.

Of a Rectangular Plane Triangle, Having the Area = 1728, and the Aggregate of the Hy-

Hypotenuſe and Cathetus = 134.533211: To find the Sides.

Of the Arithmetical Queſtions, take only theſe two following, viz.

A Ship Sails from a Port upon ſome Rumb, between the South and the Weſt, until the Diſtance Sailed be 20 Leagues more than the Departure from the Meridian of the Place, whereat the Voyage began, and 90 Leagues more than the Difference of Latitude: What is the Diſtance, Departure, Difference of Latitude, and the Courſe?

Three Merchants, *viz.* A, B, C, enter into Company, A put in 350 *l.* more than B, and B and C together put in 840 *l.* their Common Gain is 660 *l.* whereof C his ſhare is 210 *l.* What diſtinct Stock did each Merchant bring into the Bank, and what particular Shares of the Common Gain muſt A and B receive?

A SYNOPSIS OR METHODICAL INDEX OF THE *Stereometrical Problems*, &c. in this BOOK.

Of a Circle.

PROBL.

Having the Diameter; To find the quantity of the Periphery, pag. 2. *And the Converse*, pag. 3. — 2.

To find the Circles Area in Square Inches, or in Wine, or Ale Gallons, pag. 3. *And the Converse*, pag. 5. — 3.

Having the Diameter of a Circle, and versed Sine of its Segment; To Calculate the Chord-line, the Periphery, or right extension of the Arch-line, Area of the Sector, and Area of the Segment, pag. 5, 7. *Also, having a Circles whole Area, in Square Inches, and the versed Sine of its Segment; To find the Area of that* — 4.

Circles Segment, by a ſingle Multiplication, with the aid of the Table of Area's of Segments, pag. 7.

5. *To find the Area of an Ellipſes Segment*, pag. 8.

6. *Having the verſed Sine and Chord of a Circles Segment; To find the Diameter of the Circle, and Area of the Segment*, pag. 9.

7. *Having the Diameter of a Circle, and Area of its Segment, either in Square Inches, or Ale or Wine Gallons; To find the quantity of the Segments Arrow*, pag. 10, 11.

8. *Having the Area of a whole Circle, and the Area of its Segment, either in Ale or Wine Gallons; To find the quantity of the Chord-line*, pag. 11.

9. *Having a Circles Diameter; and the verſed Sine of its Segment; To find the Chord line*, pag. 12.

Of an Ellipſis.

11. *To Gauge and Inch a Tunn or Veſſel, having parallel an equal Elliptical Baſes, alike ſituate*, pag. 15, 109, 238. *And not alike ſituate*, pag. 239, 240.

12. *To Gauge a Tunn conſiſting of parallel and unequal Elliptical Baſes, alike ſituate*, pag. 17, 147. *And not alike ſituate*, pag. 244. *Whether the ſame be a Fruſtum of an Elliptical Cone at* pag. 241. *Or a Cylindroid, by Calculation*, pag. 148, *Or by the Cyclometrical Table*, pag.

pag. 242, *In whole*, pag. 147, *And gradually from Inch to Inch*, pag. 149.

To Gauge a Tunn in the form of a Circulo-Elliptical Solid, called a Cylindroid, *in whole, by Calculation*, pag. 19, 154, 156. *And gradually from Inch to Inch*, pag. 158. *And by the Cyclometrical Table*, pag. 111, 246. 13.

Of the first Ten Regular POLYGONS, or POLYGONIAL SOLIDS.

1. *Having the Side of an Æquilateral Triangle; To find by a ſingle Multiplication, (without knowledge of the Perpendicular) the Triangles Area, either in Square Inches, Acres, Ale Gallons, or Beer Barrels, applicable to the Gauging (either in whole, or in part) of a Tunn, in the form of an Æquilateral Triangular Solid, when the Baſes are equal, parallel, and alike ſituate*, pag. 1. *To Gauge ſuch a Tunn, when Pyramidal*, (i. e.) *the Baſes being unequal, but parallel, and alike ſituate, in whole*, pag. 261, 256. *And gradually, from Inch to Inch*, pag. 262. 1.

2. *Having the length, breadth, and depth of a Square Tunn, conſiſting* (1.) *of equal Baſes, parallel, and alike ſituate; To Gauge and Inch the ſame*, pag. 21, 22. (2.) *If the Tunn conſiſt of unequal Baſes, and proportional Sides, parallel and alike ſituate, being the Fruſtum of a Pyramid; To Gauge the whole Tunn*, pag. 169. 14.

And to Inch the ſame, pag. 170. (3.) *If the Tunn conſiſt of unequal Baſes, and diſproportional Sides, parallel and alike ſituate, call'd a* Priſmoid; *To Gauge the ſame*, pag. 148, 247. *To Inch it*, pag. 149, 248. *When the Sides are not alike ſituate*, pag. 250.

54. (4.) *If the Tunn conſiſt of two parallel, quadrangular and unequal Baſes, but equal ſides to each Baſe, it is the Fruſtum of a Pyramid. Gauged*, pag. 162. *And Inched*, pag. 163. *But after the beſt Form*, pag. 168.

(5.) *If the Tunns Baſes be Heterogeneal, viz. that at the bottom, a perfect Square, and the above, a Rectangular Parallelogram, to Gauge the whole Tunn*, pag. 251. *And to Inch it, Rule in* pag. 158.

Having the
15. *Dimenſions of a* Pentagonal, pag. 24, 257, 26
16. *Tunn, in the* Hexagonal, pag. 25, 258.
17. *form of any of* Heptagonal, pag. 26, 258.
18. *theſe eight Po-* Octagonal, pag. 27, 258.
19. *lygonial Solids*; Nonagonal, pag. 28, 258.
20. *To Gauge and* Decagonal, pag. 29, 259.
21. *Inch the ſame*; Undecagonal, pag. 30, 25
22. *when the Tunns Baſes are* =, *or* Dodecagonal, pag. 31, 25

When the Tunn is Pyramidal, pag. 256, 26

23. *Having the Side of a Tunn, in the form of any of theſe Polygonal Solids; to find its Diameter*, pag. 32.

10. *To Inch a Cooling Back in the form of a Rhomboidal Solid*, pag. 13.

As alſo, to Gauge the Wort remaining in it, when partly empty, pag. 127. 47.

A Cyclometrical Table, exhibiting the Area's of all Circles (in Ale Gallons, &c.) *from* 1 *Inch to* 6 *Yards Diameter*; *and in* (*Wine Gallons*, &c.) *from* 1 *Inch, to* 5 *Feet Diameter*; *both Calculated to every* $\frac{1}{10}$ *and quarter of an Inch*, pag. 35, 50.

The Conſtruction of this Table appears in the Rule upon Probl. 3. *but more particularly and fully in* pag. 233. *Some of its excellent Utility and various Uſes appear* (inter alia) *in* pag. 235, 67, 68, 89, 93, 94.

A compendious way for Gauging and Inching five ſorts of Tunns, by aid of this Cyclometrical Table;

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2. Cylindrical, pag. 237.
3. Spheroidal, pag. 238.
4. Elliptical, *of equal, or unequal Baſes*, pag. 238.
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Having the Diameter of the Baſe of a Sphears Section (*intercepted*, &c.) *and the Sections Altitude*; *to find the whole Axe*, pag. 56. 25.

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Ha-

26. *Having a Sphears Axe and the Altitude its Section, (intercepted as aforeſaid) to find t Diameter of that Sections Baſe*, pag. 57.

27. *Or to find that Sections liquid Capacity, wit out knowledge of the Diameter of its Baſe*, p. 5

29. *Having the Axe of a Sphear, and Altitude its Zone, (intercepted between two Planes p rallel, one of them paſſing by the Center) to ther with the Zones leſſer Baſe; to find the Zon liquid Capacity by Calculation, and by the C clometrical Table*, pag. 62.

30. *To find the Side of a Cube, equal (in Cap city) unto that of a Globe propoſed*, pag 63.

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31. *Having the Diameter of the Baſe, and titude; to Gauge and Inch the ſame by Cal lation*, pag. 65. *And by the Cyclometrical Tab* pag. 66.

32. *Having a Cylindrical Veſſels liquid Capaci to find the Diameter of its Baſe, by Calculat* pag. 67. *And by the Cyclometrical Table*, p.6

33. *Having a Cylindrical Tunns liquid Capaci and the Diameter of its Baſe; to find its Dep or Altitude, by the Cyclometrical Table*, pag. 6

34. *To Gauge and Inch the Fruſtum of a Cylind cut with a Plane, parallel to the Axe, by aid the Table of Area's of Segments*, pag. 70.

Of a Cylinders Fruſtum cut with a Plane, p rallel to the Axe, (the Axe lying parallel to

Horizon) having the Diameter of the Cylinders Baſe, the liquid Capacity of the whole Cylinder, and of its Fruſtum; to find the quantity of D, m, *or* y, m, *either of the Fruſtums Altitude (in Inches, &c.) which is the Arrow, or the Diameters dry or wet part,*

Of a Cone.

Having the Axe or Altitude, together with the Diameter of its Baſe; to find its liquid Capacity by Calculation, pag. 72. *And by the Cyclometrical Table,* pag. 73. 35.

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Having the extream Diameters and Depth; to find the whole Cones Axe, pag. 74. 36.

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As alſo, to Gauge the ſame, Inch by Inch, from the leſſer Diameter to the greater, by Calculation, pag. 77. 38.

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To find any Tunns Dripp or Fall, pag. 87. 40.

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44. *Having the perpendicular Altitude, and Side-line of a Conical Tunn, together with its Diagonal; to find the Vessels extream Diameters*, &c. pag. 115.

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Of a Sphæroid.

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THE SECOND PART.

The Art of Wine, Brandy, or Oyl Cask Gauging, explained in Fourteen Problems, *Viz.*

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56. *Having a Casks Boung Diameter and Diagonal; To find the Length Arithmetically*, pag. 176.

57. *Having a Casks Dimensions; To find the Quantity of the Spheroids Axe*, pag. 176.

58. *Having a Casks Dimensions; To find its whole liquid capacity, as Spheroidal*, pag. 179, 235. *As Parabolical*, pag. 181. *As Conical*, pag. 183. *As Cylindrical*, pag. 184.

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ER-

ERRATA.

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STEREOMETRY,

OR

PRACTICAL GAUGING,

Performed

By MULTIPLICATION only.

I. PROBL.

THere is a Tun in the form of an equilateral Triangle, each of the three ſides, as well at the top as bottom, being 160 *Inches; How many ſquare Inches are this Triangles Area, and how many Beer Barrels may each wet Inch of it contain?*

The Rule.

Multiply the Square of the ſemiſide by the ſquare Root of 3, which is 1.73205—, and that Product is = the ſquare Inches of that

Triangles Area: But to find each wet Inches capacity of a Vessel in that form; in Ale Gallons, multiply the square of the semiside by .006142027— and for the Inches capacity in Beer Barrels, multiply the q. of the semiside by .0001706119+.

But if the Area be sought in Acres, multiply the q. of the semiside always by .01082532, and that Product is your desire.

Multiplicat. for Beer Bar. =.0001706119+
q. of Semiside = 6400
68244760
10236714
Resp. Beer Barrels 1.0919161600
Per each wet Inch.

1.73205—
q. of semiside =6400
69282000
1039230
Square Inches 11085.12000
Area of equilateral Triangle.

II. Probl.

The Diameter of a Circle is =1.5957691— *Inch. What is its Periphery?*

The

The Rule.

Mult. the Diam. always by = 3.1415927+; Area of that Circle whose Diameter = 2, and that Product, *Resp.* = 5.01325624 the Circles Periphery sought.

The Converse.

The Periphery of a Circle is = 5.01325624+
What is its Diameter?
Multiplicator for 3.1415927 = .31831+

501325624
1503976872
4010604992
501325624
1503976872

Respons. = 1.595769593754 4
The Diameter *prout supra.*

III. Probl.

There is a Circle, whose Diameter is = 200 *Inches; how many square Inches are* = *this Circles Area, and also what is the Area of this Circle in Ale Gallons?*

The Rule.

Multiply the q. of the Diameter given, by the Area of that Circle, whose Diameter is unity, *viz.* by .7853981635+ and that Product is = Area of the Circle proposed, in square Inches, Perches, *&c.* But if the Circles Area be sought in Ale Gallons, Multiply the q. of

the given Diameter by .0027851+, and for Wine Gallons by .0011333 31+, and that Product is your desire; or it appears by ocular inspection of the Cyclometrical Table.

.7853981635
q. of Diameter 40000
Area in square Inches 31415.9265400000

Multiplicator for Ale Gal. = .0027851+
40000
Area in Ale Gallons 111.4040000

The Converse.

There is a CIRCLE whose AREA is =31415.92654 *square Inches, or* 111.404 *Ale Gallons, what is its Diameter?*

The Rule.

Multiply the square Inches of the Area by 1.273239549, (the q. of the Diameter of that Circle, whose Area is unity;) which done, the square Root of that Product is = the Circles Diameter sought.

But if the Area be proposed in Ale Gallons, multiply the same by 359.0536+, and if in Wine Gallons, by 294.1183+, and the square Root of that Product is = the Circles Diameter sought.

31415.93+

```
                    31415.93+
                     1.27324+
                    ---------
                    12566372
                     6283186
                    9424779
                   21991151
                   6283186
                  3141593
                  ---------
q. of 200 the Diam. 40000.0187132
```

```
Multiplicator for Ale Gal. = 359.0536+
                             111.404
                            ---------
                            14362144
                           143621480
                            3590537
                           3590537
                          3590537
                          ---------
                          40000.0183944
```

The q. of 200 the Diameter sought.

But having a Circles Area in Gallons, its Diameter is given by inspection of the Cyclometrical Table.

IV. Probl.

To Calculate the Area of a Sector, and of the Segment of a Circle. *Fig. in Tab. of Areas of Segm. of a Circle.*

The Semidiameter of the Circle R C, (the whole Circles Area being unity)
=.5641 8958—

The verſed Sine R *o* in the Ratio of .3620 parts, but is really =.40847326

And *o* C =.15571632

whole Diameter = 1.12837917
.3620

Product =.40847326+

Rem. of Diam. =.71990590 — 9,8572757
Verſed Sine R *o* =.40847326 — 9,6111635
Then a Geomet. mean proport. — 9,4684392
Between them
= Semichord, *m o* .542275 — 9,7342196

To find the Angle at the Center, viz. *the Angle m C n, the Analogiſm is,*

Compl. Ar.	
—0,2485750 :	As C *n* which is = ſemidiam. .5641 8958—
. . . . . :	to the Radius, or Sine of 90 d. the right Angle C *o n:*
—9,7342196 :	So is *o n* the Semi-Chord .542275
9,9827946 :	to the Sine of deg. 73,97839 2

The Angle at the Center *m* C *n* deg. 147,95678

The

The perpendic. of Triangle C o = .15571632
m o the Triangles Base = .542275
Product .084441067428oo
Area of Triangle *m* C *n o*.

To find the Periphery of the Arch Line, m R *n, use this Analogism.*

Compl. Ar.
7,4436975 : As 360 degrees
2,1701348 : to 147.95678 Ang. at the Cen.
0,5496049 : So 3.544908, the Periph. of the same Circ. whose Area is unity
0,1634372 : to 1.456925 = Periphery of Arch Line *m* R *n*.

.728462 5 the Arches Semiperiphery.
.5641895 8 the side of the Sector C *n* = Radius of the Circle.
.41099095192075 Product, = Area of the Sector, *m* R *n* C.
.084441067428oo = Area of Triangle, *m* C *n o*.
.3265498844927 5 = Area of Segm. *m* R *n o*.
And not
.32718,66756
nor
.32618, *&c.*

Suppose the Circles Diameter be 25 *Inches*, *and x o the versed Sine be* 9 $\frac{1}{20}$ *or* 9:05, *and the Circles whole Area* =490.8739 : *what is the Area of the Segment c x d o?* Fig. 11.

The verſed Sine = 9.95
Multiplicator for Diam. 25 = .04
.3620 = *per* .T.

Area of Circle 490.8739
.3620 *per* Tab. = .32655
24543695
24543695
29452434
9817478
14726217
Reſponſ. Area of Segment = 160.294872045

V. Probl.

Fig. 12. *Having the leſſer Diameter of an Ellipſis (ſuppoſed to be inſcribed in a Circle) to find the Area of the Ellipſes Segment, correſpondent to any Segments Area of the Circle propoſed.*

Admitting the leſſer Diameter of the Ellipſis be .85, *to find the Area of the Segment of the Ellipſis, correſpondent unto that of the Circle laſt found, ſave one.*

Compl.

Compl. Ar.

9,9475449 : As 1.12837917 the Diameter of the Circle,

—9,9294189 : to .85, the leſſer Diameter of the Ellipſis :

—9,5139495 : So is .32654988, the Area of the Segment of the Circle,

—9,3909133 : to .2459876—, the correſp. Area of the Ellipſes Segment ſought.

VI. Probl.

Having the Chord of a Circle c o d 120 *Inches, and the verſed Sine* x o 20 *Inches; to find the Diameter of the Circle, and Area of its Segment.* *Fig.* 11.

The Rule.

The Square of the Semichord *c o* or *o d*, divided by the verſed Sine *o x*, quotes the remainder of the Circles Diameter, which added unto *o x*, the verſed Sine, that Sum is = the Circles Diameter ſought: Then proceed according to direction of the precedent Rule, for finding the Segments Area.

x

q. of Semichord *c o* = 3600 (180 rem. of Diam.

x o = 2000 20 *x o*

2*x* 200 = Circ. Diam.

VII.

VII. Probl.

Fig. 11. *Having the Area of the Segment of a Circle* =779.0665 *ſquare Inches, as* c x d o, *together with the Circles Diameter* e m 200 *Inches; To find the quantity of the Arrow, or verſed Sine,* x o.

The $\frac{1}{4}$ of the Area of that Circle whoſe Diameter is 2, =.7853981635
The q. of the given Diameter 40000
Area of propoſed Circ. = 31415.92654

For which the Multiplicator =.0000318309

Propoſed Area of Seg. in q. Inch. 779.0665
Mult. for Area whole Circle =.0000318 31+

Product = .02480
.T. diff. .02450
diff. .00030

I find .02450 = *per* Table .0600, diff. .00060
Then
.00060 : 10 :: .00030 = .0005
ſo I find .02480 = .0605 Arrow of .T. .0605
The propoſed Diameter 200
Reſponſ. the verſed Sine = 12.1000

Having

Having the Area of a Circles Segment in Ale Gallons viz. 100.8652, *together with the Circles Diameter* = 400 *Inches*; *To find the quantity of the Arrow, or verſed Sine, of that Segment.* Fig. 11.

The Area of that Circle whoſe Diameter is 1;
= Ale Gallons .0027851+
q. of Diameter 400 = 160000
167106
27851
Area of whole Circle in Ale Gal. 445.616

For which the Multiplicator = .002244085

Area of Segm. propoſed in Ale Gal. 100.8652

Product = .22635
I find *per* Table = 22578 = .2770 diff. 114
diff. 57 5
As d. 114 : 10 :: 57 : 5 ſo 22635 = .2775

The given Diameter 400
Reſponſ. the verſed Sine *x o* = 111(0000

VIII. Probl.

Having the Area of the Segment of a Circle in Ale Gallons 100.8652, *together with the whole Circles Area in Ale Gallons* 445.616; *To find the quantity of the Chord Line*, c o d. Fig. 11.

Mul-

Multiplicat. for Ale Gallons always 359.0536
Whole Circles Area 445.616
Product = 160000, *&c.*
The q. of 400 = the Circles Diameter.

Then by the next precedent Rule, the verſed Sine, *x o*, = 111 Inches.

Then the Semichord c o, *being a Geometrical Mean proportional between the verſed Sine,* (*found* 111) *and* 289 *the Remainder of the Diameter, To find that Chord.*

The Remainder of the Diameter	289
The verſed Sine	111
	289
	289
	289
	32079

The q. of 179.1061 Inch. for the Semichord *c o.*

2

Reſp. 358.2122 for the Chord *c o d.*

IX. Probl.

Fig. 11. *Having a Circles Diameter* e m 200 *Inches, and the Verſed Sine of its Segment* x o 20 *Inches; To find the quantity of the Chord-line* c o d.

whole

whole Diameter = 200.
Verſed Sine = 20
Rem. Diam. = 180
Multiply *per* Verſed Sine 20
3600

Which is the q. of the Semichord *c o* 60 Inches, ſo the Chord-line is = 120 *Inches.*

X. Probl.

There is a Brewers Cooling Back in the form of a Trapezium, as a e 152 *Inches,* e i 350 *Inches,* i o 182 *Inches, and* o a 260, *and the Diagonal* o e = 320 *Inches, dividing the Trapezium into two Triangles; To find the Area of the Trapezium in ſquare Inches, as alſo to find how many Beer Barrels each wet Inch of the Back contains.* Fig. 16.

The Rule.

1. From the Semiperimeter of a Triangle ſubduct each ſide ſeverally.

2. Multiply the Semiperimeter and the three differences aforeſaid by each other, according to continual Multiplication.

3. The ſquare Root of that Product is = the Triangles Area in ſquare Inches.

4. The Sum of the Triangles Areas (ſo found) is = the Area of the Trapezium in ſquare

square Inches; which for Ale Gallons multiplied by .0035461+, and for Beer Barrels multiplied by .0000985027+, produces the capacity of each wet Inch in Ale Gallons or Beer Barrels.

$o\,a = 260$
$a\,e = 152$
$e\,o = 320$
Perimeter of Tr. $a\,e\,o = 732$
Semiperimeter $= 366 : 2{,}5634811$
$106 : 2{,}0253059$
$214 : 2{,}3304138$
$46 : 1{,}6627578$
$8{,}5819586$
$19542.46 = 4{,}2909793$

$e\,i = 182$
$i\,o = 350$
$o\,e = 320$
Perimeter of Tr. $e\,i\,o = 852$
Semiperimeter $= 426 : 2{,}6294096$
$244 : 2{,}3873898$
$76 : 1{,}8808136$
$106 : 2{,}0253059$
$8{,}9229189$
$= 4{,}4614594$

4,4614594 : 28937.40
19542.46

Area of a Trapezium in q. Inches 48479.86
Mult. for Beer Barrels = .00009850279+

43631874
33935902
9695972
242399300
38783888
43631874

Beer Barr. *per* wet Inch 4.775401468809 4
Being = the Trapeziums Area in Beer Barrels.

XI. Probl.

There is a Flote or Veſſel in the form of an Ellipſis, its Axe or greater Diameter is 51.5 *Inches, the leſſer Diameter is* 42.5 *Inches, and the depth* = 13.5 *Inches: How many Ale Gallons may each wet Inch contain, and how many ſuch Gallons is the Veſſels whole capacity?*

The Rule.

Multiply the Rectangle of the greater and leſſer Diameters by .0027851, and that Product is the Area of the Ellipſis in Ale Gallons, which is alſo equal to what each wet Inch of it contains: And the ſaid Rectangle multiplied by .003399992 produces the Ellipſes Area, or what

what each wet Inch of it contains in Wine Gallons; which multiplied by the depth, gives the whole capacity, and in like manner, the capacity at any depth propoſed.

51.5	2188.75
42.5	.0027851
2575	218875
1030	1094375
2060	1751000
2188.75	1532125
	437750

Reſp. per Inch Ale Gal. 6.095887(625
= Ellipſes Area.

The whole depth	13.5
	30479440
	18287664
	6095888
Veſſels whole capa. in Ale G.	82.294488.

XII.

XII. Probl.

There is a Flote or Veſſel in the form of an Irregular Ellipſis, whereof the Axe or greater Diameter at the top is 42 *Inches, its leſſer Diameter at the top is* 35; *its greater Diameter at the bottom is* 39.1, *and the leſſer Diameter below* = 31.8 *Inches, and the depth* 15: *How many Ale Gallons is this Veſſels capacity?*

The Rule.

Take two Geometrical Mean Proportionals; the one between the Axes or greater Diameters, and the other between the leſſer Diameters given; ſo have you the Diameters of the Mean Ellipſis; and therewith proceed for the whole capacity according to the precedent Rule for a Regular Ellipſis.

39.1
42
782
1564
1642.2 : 32154260
1,6077130 : 40.52407+

31.8
35
1590
954
1113 : 3,0464952
1,5232476 : 33.36165—

Mean Ellipſes ſhorter Diameter 33.36165—
The Mean Ellipſes Axe 40.52407+

33353155
133446600
6672330
16680825
133446600
1351.94983(99155

Multiplicator for Ale G. .0027851

135194984
675974920
10815598 72
946364888
27038996 8
3.765315499384
15 the depth.
188265775
37653155
Reſponſ. 56.4797325 Ale Gall.

XIII. Probl.

There is a Veſſel, Circular below, and Elliptical above; the Diameter of the Circular Baſe, is 40 *Inches; the Axe, or greater Diameter of the Elliptical Baſe, is* 45 *Inches; and the leſſer Diameter,* 43; *and the Veſſels depth,* 30 *Inches; How many Ale Gallons is this Veſſels Capacity?* Fig. 15.

The Rule.

1. The Rectangle of the Axe and leſſer Diameter of the Ellipſis, multiplied (for Wine Gallons) by .003399992, and (for Ale Gallons) by .0027851, produces the Area of the Elliptical Baſe in Wine or Ale Gallons.

2. The Area of the Circular Baſe, either in Wine or Ale Gallons, is given *per* Cyclometrical Table.

3. The Geometrical Mean Proportional between thoſe Areas, being multiplied by the Veſſels depth, produces its whole Capacity in Wine or Ale Gallons, according to the Multiplicator made uſe of.

45
43
135
180
1935

The Multiplicator for Ale Gallons
= .0027851
1935
139255
83553
250659
27851
5.3891685 = Area of Elliptic. Bu
4.4561598 = Area of Circle, Dian
431133480 40 Inches,
485025165 Cyclom. Tabl
269458425
53891685
323350110
269458425
215566740
215566740
24.014996025 12630 } 1,3804825
0,6902413

0,6902413 Mean Area = 4.90051 Ale G
30 Veſſ. de
Reſponſ. Ale Gal. 147.01530 the Ve
ſels whole Capaci

XIV. Probl.

There is a ſquare Tunn, whereof the length is 198 *Inches, the breadth* 188.5, *and the depth* 60 *Inches: How many Beer Barrels, and how many Gallons is the whole Tunns liquid Capacity? and how many of each doth each wet Inch of it contain?*

The Rule.

Multiply the length and breadth together, and that Product multiply by .0035461+ being the natural Number, proper to the Complement Arithmetical of the Logarithm of 282 (the Cubical Inches in a Beer or Ale Gallon) which done, this laſt Product ſhews the Tunns Area in Beer Gallons, being always equal unto the number of Beer Gallons, which each wet Inch of it may contain; which multiplied again by any number of wet Inches propoſed, produces the number of Gallons, remaining in the Tunn at that altitude of Liquor inquired of. The like Multiplicator for Beer Barrels is =.00009850279+ and for Ale Barrels =.0001108153.

The breadth 188.5
The length 198

15080
16965
1885

37323

The Multiplicator for Ale Gallons .0025461

37323
223938
149292
186615
111969

Ale Gall. upon each wet Inch 132.3510903
The depth 6

Ale Gallons 7941.0654185

The Tunns whole Capacity being 220 Bee Barrels, a Kilderkin, 3 Gallons, and half Pint, *proximè.*

ANOTHER INSTANCE.

A ſquare Tunns length is 141.5 *Inches, th breadth* 118, *and the depth* 55.9 *Inches: How many Beer Barrels (at* 36 *Gallons to the Barrel is the whole Tunns Capacity, and how much co tains it upon each wet Inch?*

The Rule.

Multiply the length and breadth together and that Product multiply by .0000985028+ being

being the natural Number proper to the Complement Arithmetical of the Logarithm of 10152 (the Cubical Inches in a Beer Barrel) which done, that Product shews the Tunns Area in Beer Barrels, which is always = the number of Beer Barrels, or parts of a Beer Barrel, that each wet Inch of the Vessel proposed, may contain.

The length 141.5
The breadth = 118

11320
1415
1415
166970

Multiplicator for Beer Barrels = .000098502 8

1335760
333940
8348500
1335760
1502730
16447012516 0

Resp. 1.644701 being 1 $\frac{1}{2}$ Barrel, and 5 Gallons, and 1 Quart *ferè*, *per* each wet Inch.

Tunns Area = Ale Gallons 1.6447—
Its whole depth 55.9

148023
82235
82235

Resp. Tunns whole Capac. in B. B. 91.93873
which is 91 Barrels $\frac{3}{4}$ 7 Gallons *ferè*.

XV. Probl.

Fig. 6. *There is a Tunn, whose Base or Superficies is in the form of a Pentagon, having five equal sides at the top and bottom, each being* 48 *Inches, and the depth* 59 *Inches; how many Ale Gallons may this whole Tunn contain, and how much per wet Inch?*

The Rule.

Multiply the Sq. of a Side by .00610099+ and the Product (so accruing) is always equal to the Pentagons Area in Ale Gallons, which shows also how many such Gallons it holds upon every wet Inch: But if the Square of the Side be multiplied by .0001694718, or by .0001695+, that Product shows how many Beer Barrels, or what parts of a Beer Barrel, each wet Inch may contain; the which multiplied by the whole depth, produces the Tunns whole Capacity, as also the quantity of liquor remaining in it, at any altitude of liquor assigned.

The

The ſide = 48
48
384
192
The q. of the ſide = 2304
The Multiplicator for Ale Gall. = .006101+
2304
23040
13824
Reſponſ. Ale Gallons *per* wet Inch 14.056704
The whole depth 59
126510336
70283520
Reſponſ. Ale Gallons 829.345536
The Tunns whole Capacity.

XVI. Probl.

There is a Tunn, whoſe Superficies or Baſe is in the form of an Hexagon, or ſix ſided Figure, equal at the top and bottom, each being 48 *Inches, and the depth* 59: *How many Ale Gallons may this whole Tunn contain, and how much* per *wet Inch.*

The Rule.

Multiply the Square of the ſide (if the Hexagons Area be ſought in Ale Gallons) by .009213034+, and if in Beer Barrels, by .000255917 7+

.0002559177+, the which being multiplied by any depth inquired of, produces the Capacity of the Veſſel at that altitude of liquor ſought.

Multiplicator for Ale Gallons .009213+
The Square of 48 the ſide = 2304

36852
276390
18426

Per Inch Gall. 21.226(752
The depth 59

191043
106135

Reſponſ. Ale Gallons 1252.393

XVII. Probl.

There is a Tunn in the form of an Heptagon, having ſeven equal ſides at the top and bottom, each being 48 Inches, and the depth 59 Inches: How many Ale Gallons may each wet Inch of this Tunn hold, and how many may the whole Tunn contain?

The Rule.

Multiply the Square of the ſide (if the Heptagons Area be ſought in Ale Gallons) by .0128881+,

.01288881+, and if in Beer Barrels, by .00035022 5—, the which being multiplied by any depth inquired of, produces the Veſſels Capacity in Gallons or Barrels, at that altitude of liquor propoſed.

The Multiplicator for Ale Gallons .01285+

The Square of the ſide 48 = 2304

5156
38670
2578
29.69856

The whole depth 59

26728704
14849280
1752.21504

Reſponſ. 1752 Gallons $\frac{1}{4}$ *ferè.*

XVIII. Probl.

There is a Tunn in the form of an Octogon, having eight equal ſides at the top and bottom, each being 48 *Inches, and the depth* 59: *How many Ale Gallons may each wet Inch of this Veſſel hold, and how many ſuch Gallons is the whole Tunns Capacity?*

The

The Rule.

Multiply the Square of the ſide (if the Octogons Area be ſought in Ale Gallons) by .01712208—, and if in Beer Barrels by .00047561 36, the which being multiplied by any depth inquired of, produces the Capacity of the Veſſel at the altitude of liquor ſought.

The Multiplicator for Ale Gall. =.017122—
The Square of 48 the ſide = 2304
68488
513660
34244
Reſponſ. per wet Inch Ale Gall. 39.4490(88
The whole depth = 59
3550419
1972455
The whole Tunns Capacity 2327.4969
being 2327 Gallons $\frac{1}{2}$ *proximè.*

XIX. Probl.

There is a Tunn in the form of a Nonogon, having nine equal ſides at top and bottom, each being 48 *Inches, and the depth* 59 *Inches: How many Ale Gallons may each wet Inch of this Veſſel contain, and how many ſuch Gallons is this Tuns whole Capacity.*

The

The Rule.

Multiply the Square of the ſide (if the Nonogons Area be ſought in Ale Gallons) by .021921 37+, and if in Beer Barrels by .0006089269—, the which being multiplied by any depth inquired of, produces the Tunns Capacity at the altitude of liquor inquired of.

The Multiplicator for Ale Gallons = .0219—
The Square of the ſide 48 = 2304
876
6570
438
Reſponſ. per wet Inch Ale Gall. 50.4576
The Tunns whole depth 59
4541184
2522880
The Veſſels whole Capacity 2976.9984

XX. Probl.

There is a Tunn in the form of a Decagon, having ten equal ſides at the top and bottom, each being 48 *Inches, and the depth* 59: *How many Ale Gallons may each wet Inch of this Veſſel hold, and how many ſuch Gallons is the Tunns whole Capacity?*

The

The Rule.

Multiply the Square of the ſide (if the Octogons Area be ſought in Ale Gallons) by .01712208—, and if in Beer Barrels by .00047561 36, the which being multiplied by any depth inquired of, produces the Capacity of the Veſſel at the altitude of liquor ſought.

The Multiplicator for Ale Gall. =.017122—
The Square of 48 the ſide = 2304
68488
513660
34244
Reſponſ. per wet Inch Ale Gall. 39.4490(88
The whole depth = 59
3550419
1972455
The whole Tunns Capacity 2327.4969
being 2327 Gallons $\frac{1}{2}$ *proximè.*

XIX. Probl.

There is a Tunn in the form of a Nonogon, having nine equal ſides at top and bottom, each being 48 *Inches, and the depth* 59 *Inches: How many Ale Gallons may each wet Inch of this Veſſel contain, and how many ſuch Gallons is this Tuns whole Capacity.*

The

The Rule.

Multiply the Square of the ſide (if the Nonogons Area be ſought in Ale Gallons) by .021921 37+, and if in Beer Barrels by .000608 9269—, the which being multiplied by any depth inquired of, produces the Tunns Capacity at the altitude of liquor inquired of.

The Multiplicator for Ale Gallons = .0219—
The Square of the ſide 48 = 2304
876
6570
438
Reſponſ. per wet Inch Ale Gall. 50.4576
The Tunns whole depth 59
4541184
2522880
The Veſſels whole Capacity 2976.9984

XX. Probl.

There is a Tunn in the form of a Decagon, having ten equal ſides at the top and bottom, each being 48 *Inches, and the depth* 59: *How many Ale Gallons may each wet Inch of this Veſſel hold, and how many ſuch Gallons is the Tunns whole Capacity?*

The

The Rule.

Multiply the Square of the ſide (if the Decagons Area be ſought in Ale Gallons) by .02728443—, and if in Beer Barrels by .00075790 1—, the which being multiplied by any depth inquired of, produces the Veſſels Capacity at that altitude of liquor propoſed.

The Multiplicator for Ale Gallons = .0273+
The Square of 48 the ſide = 2304

1092
8190
546

Reſp. per wet Inch Ale Gall. 62.8992
The Tunns whole depth 59

566092 8
3144960

The whole Tunns Capacity = 3711.0528
being 3711 Gallons, and nigh half a Pint.

XXI. Proel.

There is a Tunn in the form of an Undecagon, having eleven equal ſides at top and bottom, each being 48 *Inches, and the depth* 59 *Inches: How many Ale Gallons may each wet Inch of this Veſſel contain, and how many ſuch Gallons is this Tunns whole Capacity?*

The

The Rule.

Multiply the Square of the ſide (if the Undecagons Area be ſought in Ale Gallons) by .03321148—, and if in Beer Barrels, by .00092254 14—, the which being multiplied by any depth inquired of, produces the Veſſels Capacity at that altitude of liquor propoſed.

The Multiplicafor for Ale Gallons = .0332—
The Square of the ſide 48 = 2304

1328
9960
664

Reſp. per wet Inch Ale Gallons = 76.4928
The Tunns whole depth 59

6884352
3824640

The whole Tunns Capacity = 4513.0752
being 4513 Gall. and above half a Pint.

XXII. PROBL.

There is a Tunn in the form of a Dodecagon, having twelve equal ſides at top and bottom, each being 48 *Inches, and the depth* 59 *Inches: How many Ale Gallons may each wet Inch of this Veſſel contain, and how many ſuch Gallons is the Tunns whole Capacity?*

The

The Rule.

Multiply the Square of the ſide (if the Dodecagons Area be ſought in Ale Gallons) by .03970267+, and if in Beer Barrels, by .001102852—, the which being multiplied by any depth inquired of, produces the Veſſels Capacity in Gallons or Barrels at that altitude of liquor inquired of.

The Multiplicator for Ale Gall. = .039703+
The Square of 48 the ſide = 2304
158812
1191090
79406
Reſponſ. per wet Inch Ale Gall. 91.4757(12
The Tunns whole depth 59
823813
4573785
The whole Tunns Capacity 5397.0663
which is 5397.0663 Gall. and half a Pint, *quam proxime.*

XXIII. Probl.

Fig. 6. *Having the Dimenſions,* viz. *the ſide and depth of a Tunn or Veſſel, in any of theſe Polygonal Forms,* viz. *in the form of a Pentagon, Hexagon, Octogon,* &c. *or any of the reſt, To find the quantity*

quantity of the Tunn or Veſſels Diameter in Inches.

The Rule

Firſt find (according to precedent inſtruction) the Area of the Tunn in Ale Gallons, being = unto what each wet Inch of it contains, which being found, multiply by 282 (the number of Cubical Inches in a Gallon) and that Product gives the Tunns Area in Square Inches. Secondly, multiply the number of Inches in each ſide by the number of the ſides, whether 5, 6, 7, 8, 9, 10, 11 or 12, and the half of that Product is = the Semiperimeter of the Veſſel. Thirdly, the ſquare Inches in the Veſſels Area, divided by its Semiperimeter, quotes the Tunns Semidiameter; which doubled is its Diameter ſought.

As for INSTANCE.

There is a Tunn in the form of a Dodecagon, having twelve equal ſides at top and bottom, each being 48 *Inches, and the depth* 59 *Inches; of how many Inches is this Veſſels Diameter?* Fig. 6.

Each ſide	48 Inches.
Number of ſides	12
The Perimeter	576
The Semiperimeter =	288

Ale Gallons.

The Tunns Area before found = 91.4757

282

1829514

7318056

1829514

Its Area in Square Inches = 25796.1474

Multiplicator for 288 Semiper. = .0034722

515922948

515922948

1805730318

1031845896

773884422

89.569383(00228

2

Respons. Diameter 179.138766

being = 179.$\frac{14}{100}$ Inches.

A

CYCLOMETRICAL TABLE,

Exhibiting

The AREA's of CIRCLES

IN

ALE GALLONS

AND

DECIMILLESSIMAL PARTS,

Calculated to every Tenth Part and Quarter of an Inch of the Diameter of the CIRCLE, from One Inch, unto Six Yards Diameter, according to 282 Cubical Inches to such Gallon.

Di. Inches.	.0	.1	.2	.25	.3	.4
1	0.0028	0.0034	0.0040	0.0044	0.0047	0.0055
2	0.0111	0.0122	0.0134	0.0140	0.0147	0.0160
3	0.0251	0.0268	0.0286	0.0294	0.0304	0.0322
4	0.0446	0.0469	0.0492	0.0504	0.0516	0.0540
5	0.0696	0.0724	0.0753	0.0767	0.0782	0.0812
6	0.1003	0.1037	0.1072	0.1089	0.1107	0.1142
7	0.1365	0.1404	0.1444	0.1464	0.1484	0.1525
8	0.1782	0.1826	0.1871	0.1894	0.1917	0.1964
9	0.2256	0.2306	0.2357	0.2383	0.2409	0.2461
10	0.2785	0.2841	0.2898	0.2927	0.2955	0.3013
11	0.3370	0.3431	0.3493	0.3524	0.3555	0.3618
12	0.4011	0.4079	0.4147	0.4181	0.4215	0.4284
13	0.4707	0.4780	0.4853	0.4896	0.4927	0.5001
14	0.5459	0.5537	0.5616	0.5655	0.5695	0.5775
15	0.6266	0.6349	0.6433	0.6476	0.6518	0.6604
16	0.7130	0.7219	0.7309	0.7354	0.7399	0.7490
17	0.8049	0.8144	0.8239	0.8287	0.8335	0.8432
18	0.9024	0.9125	0.9226	0.9277	0.9327	0.9429
19	1.0054	1.0160	1.0267	1.0321	1.0374	1.0482
20	1.1140	1.1251	1.1363	1.1419	1.1476	1.1590
21	1.2282	1.2399	1.2517	1.2577	1.2636	1.2755
22	1.3480	1.3603	1.3727	1.3789	1.3851	1.3975
23	1.4733	1.4861	1.4990	1.5055	1.5120	1.5250
24	1.6042	1.6176	1.6310	1.6378	1.6445	1.6581
25	1.7407	1.7546	1.7686	1.7757	1.7827	1.7968
26	1.8827	1.8972	1.9118	1.9191	1.9264	1.9411
27	2.0303	2.0454	2.0605	2.0681	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2227	2.2305	2.2463
29	2.3423	2.3585	2.3747	2.3828	2.3910	2.4073
30	2.5066	2.5233	2.5401	2.5485	2.5569	2.5738
31	2.6765	2.6938	2.7112	2.7199	2.7286	2.7461
32	2.8519	2.8697	2.8876	2.8965	2.9055	2.9236
33	3.0330	3.0514	3.0698	3.0791	3.0883	3.1069
34	3.2196	3.2386	3.2576	3.2672	3.2767	3.2958
35	3.4117	3.4312	3.4508	3.4606	3.4704	3.4901

.5	.6	.7	.75	.8	.9
0.0063	0.0072	0.0081	0.0085	0.0091	0.0100
0.0174	0.0188	0.0203	0.0211	0.0218	0.0234
0.0341	0.0361	0.0381	0.0392	0.0402	0.0424
0.0564	0.0589	0.0615	0.0628	0.0642	0.0669
0.0842	0.0873	0.0904	0.0920	0.0936	0.0969
0.1177	0.1213	0.1250	0.1269	0.1288	0.1326
0.1567	0.1609	0.1652	0.1674	0.1695	0.1738
0.2012	0.2060	0.2109	0.2134	0.2158	0.2207
0.2514	0.2567	0.2620	0.2647	0.2674	0.2729
0.3071	0.3130	0.3189	0.3219	0.3249	0.3309
0.3682	0.3746	0.3811	0.3844	0.3877	0.3944
0.4352	0.4422	0.4492	0.4527	0.4563	0.4635
0.5076	0.5151	0.5227	0.5266	0.5304	0.5381
0.5856	0.5937	0.6019	0.6060	0.6101	0.6183
0.6691	0.6778	0.6865	0.6909	0.6953	0.7041
0.7582	0.7674	0.7767	0.7814	0.7860	0.7954
0.8529	0.8627	0.8725	0.8774	0.8824	0.8924
0.9532	0.9636	0.9740	0.9792	0.9844	0.9949
1.0590	1.0699	1.0809	1.0864	1.0919	1.1029
1.1704	1.1818	1.1933	1.1991	1.2049	1.2165
1.2874	1.2994	1.3114	1.3175	1.3236	1.3357
1.4100	1.4225	1.4351	1.4414	1.4478	1.4605
1.5381	1.5512	1.5644	1.5710	1.5776	1.5909
1.6718	1.6855	1.6992	1.7061	1.7130	1.7268
1.8110	1.8252	1.8395	1.8467	1.8538	1.8682
1.9558	1.9706	1.9854	1.9929	2.0003	2.0253
2.1062	2.1215	2.1369	2.1747	2.1524	2.1679
2.2622	2.2781	2.2941	2.3021	2.3101	2.3262
2.4237	2.4402	2.4567	2.4650	2.4733	2.4899
2.5908	2.6078	2.6249	2.6335	2.6420	2.6592
2.7635	2.7811	2.7987	2.8076	2.8164	2.8341
2.9418	2.9600	2.9782	2.9873	2.9964	3.0147
3.1256	3.1443	3.1630	3.1724	3.1818	3.2007
3.3150	3.3342	3.3535	3.3632	3.3728	3.3922
3.5099	3.5297	3.5496	3.5595	3.5695	3.5895

Di. Inches	0	.1	.2	.25	.3	.4
36	3.6095	3.6296	3.6497	3.6598	3.6699	3.690
37	3.8128	3.8335	3.8542	3.8645	3.8749	3.895
38	4.0217	4.0429	4.0642	4.0748	4.0855	4.106
39	4.2361	4.2579	4.2797	4.2906	4.3016	4.323
40	4.4562	4.4785	4.5009	4.5121	4.5233	4.545
41	4.6818	4.7047	4.7276	4.7391	4.7506	4.773
42	4.9129	4.9363	4.9598	4.9716	4.9833	5.006
43	5.1497	5.1737	5.1977	5.2098	5.2218	5.245
44	5.3920	5.4166	5.4412	5.4535	5.4658	5.490
45	5.6398	5.6649	5.6901	5.7027	5.7153	5.740
46	5.8933	5.9189	5.9446	5.9575	5.9704	5.996
47	6.1523	6.1785	6.2048	6.2180	6.2311	6.257
48	6.4169	6.4437	6.4705	6.4840	6.4974	6.524
49	6.6870	6.7143	6.7417	6.7554	6.7691	6.796
50	6.9628	6.9907	7.0186	7.0326	7.0466	7.074
51	7.2440	7.2724	7.3009	7.3152	7.3295	7.358
52	7.5309	7.5599	7.5889	7.6034	7.6180	7.647
53	7.8233	7.8529	7.8825	7.8973	7.9122	7.941
54	8.1214	8.1515	8.1817	8.1968	8.2119	8.242
55	8.4249	8.4555	8.4862	8.5016	8.5170	8.547
56	8.7341	8.7653	8.7965	8.8121	8.8278	8.859
57	9.0488	9.0806	9.1124	9.1283	9.1443	9.176
58	9.3691	9.4015	9.4339	9.4501	9.4663	9.498
59	9.6949	9.7278	9.7608	9.7773	9.7938	9.826
60	10.0264	10.0598	10.0933	10.1101	10.1269	10.160
61	10.3634	10.3974	10.4314	10.4485	10.4655	10.499
62	10.7059	10.7405	10.7751	10.7924	10.8098	10.844
63	11.0541	11.0892	11.1244	11.1420	11.1596	11.194
64	11.4078	11.4435	11.4792	11.4971	11.5150	11.550
65	11.7670	11.8032	11.8395	11.8577	11.8759	11.912
66	12.1319	12.1687	12.2055	12.2239	12.2424	12.279
67	12.5023	12.5396	12.5770	12.5958	12.6145	12.652
68	12.5783	12.9162	12.9542	12.9732	12.9922	13.030
69	13.2599	13.2984	13.3369	13.3562	13.3755	13.414
70	13.6470	13.6860	13.7251	13.7447	13.7642	13.803

.5	.6	.7	.75	.8	.9
3.7104	3.7307	3.7511	3.7614	3.7716	3.7922
3.9165	3.9374	3.9584	3.9689	3.9794	4.0005
4.1282	4.1496	4.1711	4.1819	4.1927	4.2144
4.3455	4.3675	4.3896	4.4007	4.4117	4.4339
4.5683	4.5909	4.6135	4.6249	4.6362	4.6590
4.7966	4.8197	4.8429	4.8546	4.8662	4.8895
5.0306	5.0543	5.0781	5.0900	5.1019	5.1258
5.2701	5.2943	5.3186	5.3308	5.3430	5 3675
5.5152	5.5400	5.5648	5.5772	5.5897	5.6147
5.7659	5.7913	5.8167	5.8294	5.8422	5.8677
6.0221	6.0480	6.0740	6.0870	6.1000	6.1261
6.2839	6.3 104	6.3369	6.3502	6.3635	6.3902
6.5513	6.5783	6.6054	6.6190	6.6325	6.6597
6.8242	6.8518	6.8795	6.8933	6.9072	6.9350
7.1027	7.1308	7.1590	7.1731	7.1873	7.2156
7.3868	7.4155	7.4443	7.4587	7.4731	7.5020
7.6764	7.7057	7.7350	7.7497	7.7644	7.7938
7.9717	8.0015	8.0314	8.0463	8.0613	8.0913
8.2724	8.3028	8.3333	8.3485	8.3638	8.3943
8.5788	8.6098	8.6408	8.6563	8.6719	8.7030
8.8907	8.9222	8.9538	8.9696	8.9854	9 0171
9.2082	9.2402	9.2723	9.2884	9.3045	9.3368
9.5313	9.5639	9.5955	9.6129	9.6292	9 6620
9.8600	9.8932	9.9264	9.9431	9.9597	9 9930
10.1942	10.2279	10.2617	10.2786	10.2955	10.3294
10.5339	10.5682	10.6026	10.6198	10.6370	10.6714
10.8793	10.9141	10.9490	10.9665	10.9840	11.0190
11.2302	11.2656	11.3011	11.3188	11.3366	11.3722
11.5867	11.6226	11.6586	11.6767	11.6947	11.7308
11.9488	11.9853	12.0219	12.0402	12.0585	12.0952
12.3164	12.3535	12.3906	12.4092	12.4278	12.4650
12.6896	12.7272	12.7649	12.7838	12.8026	12.8404
13.0684	13.1066	13.1448	13.1640	13 1831	13.2215
13.4527	13.4914	13.5302	13.5497	13.5691	13.6080
13.8426	13.8819	13.9213	13.9410	13.9607	14.0002

Dr. Inches	0	.1	.2	.25	.3	.4
71	14.0397	14.0793	14.1189	14.1387	14.1586	14.1983
72	14.4380	14.4781	14.5183	14.5394	14.5586	14.5989
73	14.8418	14.8825	14.9233	14.9437	14.9641	15.0049
74	15.2512	15.2924	15.3337	15.3544	15.3751	15.4165
75	15.6661	15.7080	15.7499	15.7709	15.7918	15.8338
76	16.0865	16.1290	16.1714	16.1927	16.2139	16.2565
77	16.5129	16.5558	16.5987	16.6202	16.6417	16.6848
78	16.9445	16.9879	17.0314	17.0532	17.0750	17.1187
79	17.3828	17.4258	17.4699	17.4919	17.5140	17.5582
80	17.8246	17.8692	17.9138	17.9361	17.9585	18.0033
81	18.2730	18.3181	18.3633	18.3860	18.4086	18.4539
82	18.7270	18.7727	18.8185	18.8414	18.8643	18.9102
83	19.1866	19.2329	19.2792	19.3024	19.3256	19.3720
84	19.6517	19.6985	19.7454	19.7689	19.7923	19.8393
85	20.1223	20.1697	20.2171	20.2409	20.2646	20.3122
86	20.5986	20.6465	20.6945	20.7185	20.7425	20.7906
87	21.0804	21.1289	21.1774	21.2017	21.2260	21.2757
88	21.5678	21.6168	21.6659	21.6905	21.7151	21.7643
89	22.0608	22.1104	22.1600	22.1849	22.2097	22.2595
90	22.5593	22.6095	22.6597	22.6849	22.7100	22.7603
91	23.0634	23.1141	23.1649	23.1903	23.2157	23.2666
92	23.5731	23.6243	23.6756	23.7013	23.7270	23.7785
93	24.0883	24.1401	24.1920	24.2171	24.2439	24.2959
94	24.6091	24.6615	24.7139	24.7401	24.7664	24.8190
95	25.1355	25.1884	25.2414	25.2679	25.2945	25.3476
96	25.6675	25.7210	25.7746	25.8014	25.8282	25.8817
97	26.2050	26.2591	26.3132	26.3403	26.3674	26.4216
98	26.7481	26.8027	26.8573	26.8846	26.9120	26.9668
99	27.2968	27.3520	27.4072	27.4348	27.4625	27.5178
100	27.8510	27.9067	27.9625	27.9904	28.0183	28.0743
101	28.4108	28.4671	28.5234	28.5516	28.5798	28.6363
102	28.9761	29.0330	29.0899	29.1184	29.1469	29.2039
103	29.5470	29.6045	29.6620	29.6908	29.7195	29.7771
104	30.1234	30.1815	30.2395	30.2686	30.2976	30.3558
105	30.7057	30.7642	30.8228	30.8521	30.8814	30.9401

.5	.6	.7	.75	.8	.9
14.2381	14.2780	14.3179	14.3379	14.3579	14.3979
14.6392	14.6796	14.7200	14.7403	14.7605	14.8011
15.0458	15.0868	15.1278	15.1484	15.1689	15.2100
15.4580	15.4995	15.5410	15.5619	15.5827	15.6244
15.8758	15.9179	15.9600	15.9811	16.0022	16.0444
16.2991	16.3418	16.3845	16.4059	16.4273	16.4701
16.7280	16.7712	16.8145	16.8361	16.8578	16.9011
17.1625	17.2063	17.2501	17.2720	17.2940	17.3379
17.6025	17.6468	17.6912	17.7134	17.7356	17.7801
18.0481	18.0930	18.1379	18.1604	18.1829	18.2279
18.4993	18.5447	18.5902	18.6129	18.6357	18.6813
18.9561	19.0021	19.0481	19.0711	19.0942	19.1404
19.4184	19.4649	19.5115	19.5348	19.5582	19.6049
19.8863	19.9334	19.9805	20.0041	20.0277	20.0750
20.3598	20.4075	20.4552	20.4791	20.5030	20.5508
20.8388	20.8870	20.9353	20.9594	20.9836	21.0320
21.3234	21.3722	21.4210	21.4454	21.4699	21.5188
21.8136	21.8629	21.9123	21.9370	21.9618	22.0113
22.3093	22.3592	22.4091	22.4341	22.4591	22.5092
22.8107	22.8611	22.9116	22.9368	22.9621	23.0127
23.3176	23.3686	23.4197	23.4452	23.4708	23.5219
23.8360	23.8816	23.9332	23.9590	23.9849	24.0366
24.3480	24.4001	24.4522	24.4783	24.5044	24.5567
24.8716	24.9243	24.9770	25.0034	25.0298	25.0826
25.4008	25.4540	25.5073	25.5339	25.5606	25.6140
25.9355	25.9892	26.0430	26.0699	26.0969	26.1509
26.4759	26.5302	26.5846	26.6118	26.6391	26.6936
27.0217	27.0766	27.1316	27.1591	27.1866	27.2417
27.5732	27.6285	27.6841	27.7119	27.7397	27.7953
28.1302	28.1862	28.2423	28.2703	28.2984	23.3546
28.6928	28.7494	28.8060	28.8343	28.8627	28.9194
29.2610	29.3181	29.3753	29.4039	29.4325	29.4898
29.8347	29.8924	29.9501	29.9790	30.0079	30.0657
30.4140	30.4722	30.5305	30.5596	30.5888	30.6472
30.9989	31.0577	31.1166	31.1460	31.1755	31.2344

Di. Inches	0	.1	.2	.25	.3	.4
106	31.2934	31.3525	31.4116	31.4412	31.4708	31.5300
107	31.8866	31.9462	32.0059	32.0357	32.0656	32.1254
108	32.4854	32.5456	32.6058	32.6359	32.6661	32.7265
109	33.0898	33.1505	33.2113	33.2417	33.2721	33.3330
110	33.6997	33.7610	33.8224	33.8531	33.8838	33.9453
111	34.3152	34.3771	34.4390	34.4700	34.5010	34.5630
112	34.9363	34.9987	35.0612	35.0925	35.1237	35.1863
113	35.5629	35.6259	35.6890	35.7206	35.7521	35.8152
114	36.1952	36.2587	36.3223	36.3541	36.3859	36.4496
115	36.8329	36.8969	36.9610	36.9931	37.0252	37.0895
116	37.4763	37.5409	37.6056	37.6380	37.6704	37.7352
117	38.1252	38.1904	38.2556	38.2883	38.3209	38.3863
118	38.7797	38.8455	38.9113	38.9442	38.9772	39.0431
119	39.4398	39.5061	39.5725	39.6057	39.6389	39.7054
120	40.1054	40.1723	40.2392	40.2727	40.3062	40.3733
121	40.7766	40.8440	40.9115	40.9453	40.9790	41.0466
122	41.4534	41.5214	41.5894	41.6234	41.6575	41.7257
123	42.1358	42.2043	42.2729	42.3072	42.3415	42.4101
124	42.8237	42.8928	42.9620	42.9966	43.0312	43.1004
125	43.5172	43.5869	43.6566	43.6915	43.7263	43.7961
126	44.2162	44.2864	44.3567	44.3919	44.4270	44.4974
127	44.9209	44.9917	45.0625	45.0979	45.1334	45.2043
128	45.6311	45.7024	45.7738	45.8095	45.8452	45.9167
129	46.3468	46.4187	46.4906	46.5266	46.5626	46.6347
130	47.0682	47.1406	47.2131	47.2494	47.2857	47.3583
131	47.7951	47.8681	47.9412	47.9777	48.0143	48.0874
132	48.5276	48.6012	48.6748	48.7116	48.7485	48.8221
133	49.2656	49.3397	49.4139	49.4510	49.4881	49.5624
134	50.0093	50.0840	50.1587	50.1961	50.2335	50.3083
135	50.7584	50.8336	50.9089	50.9465	50.9842	51.0596
136	51.5132	51.5890	51.6648	51.7027	51.7407	51.8167
137	52.2735	52.3496	52.4262	52.4644	52.5027	52.5792
138	53.0394	53.1163	53.1932	53.2317	53.2702	53.3473
139	53.8109	53.8883	53.9658	54.0046	54.0434	54.1210
140	54.5880	54.6660	54.7441	54.7831	54.8222	54.9004

.5	.6	.7	.75	.8	.9
31.5893	31.6486	31.7080	31.7377	31.7675	31.8270
32.1853	32.2452	32.3052	32.3352	32.3652	32.4253
32.7869	32.8474	32.9079	32.9382	32.9685	33.0291
33.3940	33.4550	33.5161	33.5466	33.5772	33.6384
34.0068	34.0684	34.1300	34.1608	34.1917	34.2534
34.6251	34.6872	34.7494	34.7805	34.8116	34.8739
35.2489	35.3116	35.3743	35.4057	35.4371	35.5000
35.8784	35.9416	36.0049	36.0366	36.0683	36.1317
36.5134	36.5772	36.6411	36.6730	36.7050	36.7689
37.1539	37.2183	37.2827	37.3149	37.3472	37.4117
37.8001	37.8650	37.9300	37.9625	37.9950	38.0601
38.4518	38.5173	38.5828	38.6156	38.6484	38.7140
39.1091	39.1751	39.2412	39.2743	39.3073	39.3735
39.7719	39.8385	39.9051	39.9385	39.9718	40.0386
40.4403	40.5074	40.5746	40.6083	40.6419	40.7092
41.1143	41.1820	41.2498	41.2837	41.3176	41.3855
41.7939	41.8622	41.9305	41.9647	41.9989	42.0673
42.4790	42.5478	42.6167	42.6511	42.6856	42.7546
43.1697	43.2391	43.3085	43.3432	43.3780	43.4476
43.8660	43.9359	44.0059	44.0409	44.0759	44.1460
44.5679	44.6384	44.7089	44.7442	44.7795	44.8502
45.2753	45.3463	45.4174	45.4530	45.4886	45.5598
45.9883	46.0599	46.1315	46.1673	46.2032	46.2750
46.7068	46.7790	46.8512	46.8873	46.9235	46.9958
47.4310	47.5037	47.5765	47.6129	47.6493	47.7222
48.1606	48.2338	48.3071	48.3438	48.3805	48.4540
48.8959	48.9697	49.0436	49.0805	49.1175	49.1915
49.6367	49.7111	49.7856	49.8228	49.8601	49.9347
50.3832	50.4581	50.5331	50.5706	50.6081	50.6832
51.1351	51.2106	51.2862	51.3240	51.3618	51.4375
51.8927	51.9688	52.0449	52.0830	52.1211	52.1973
52.6558	52.7324	52.8091	52.8474	52.8858	52.9626
53.4245	53.5017	53.5789	53.6175	53.6562	53.7335
54.1987	54.2764	54.3542	54.3931	54.4321	54.5100
54.9786	55.0569	55.1352	55.1744	55.2136	55.2921

Di. Inches	.0	.1	.2	.25	.3	.4
141	55.3706	55.4492	55.5278	55.5671	55.6065	55.6852
142	56.1588	56.2379	56.3171	56.3567	56.3963	56.4756
143	56.9525	57.0322	57.1119	57.1518	57.1917	57.2716
144	57.7518	57.8320	57.9123	57.9525	57.9927	58.0731
145	58.5567	58.6375	58.7184	58.7588	58.7993	58.8803
146	59.3672	59.4485	59.5299	59.5706	59.6114	59.6929
147	60.1832	60.2651	60.3471	60.3881	60.4291	60.5112
148	61.0048	61.0873	61.1698	61.2111	61.2524	61.3350
149	61.8320	61.9150	61.9981	62.0397	62.0812	62.1644
150	62.6648	62.7483	62.8319	62.8738	62.9156	62.9994
151	63.5031	63.5872	63.6714	63.7135	63.7556	63.8399
152	64.3469	64.4316	64.5163	64.5587	64.6011	64.6860
153	65.1964	65.2816	65.3669	65.4096	65.4523	65.5377
154	66.0514	66.1372	66.2231	66.2660	66.3090	66.3950
155	66.9120	66.9984	67.0848	67.1280	67.1713	67.2578
156	67.7782	67.8651	67.9521	67.9956	68.0391	68.1262
157	68.6499	68.7374	68.8249	68.8687	68.9125	69.0002
158	69.5272	69.6152	69.7033	69.7474	69.7915	69.8797
159	70.4101	70.4987	70.5873	70.6316	70.6760	70.7648
160	71.2986	71.3878	71.4770	71.5216	71.5663	71.6556
161	72.1926	72.2823	72.3721	72.4170	72.4619	72.5518
162	73.0922	73.1825	73.2728	73.3180	73.3632	73.4536
163	73.9973	74.0881	74.1790	74.2244	74.2699	74.3609
164	74.9080	74.9994	75.0908	75.1365	75.1823	75.2739
165	75.8243	75.9162	76.0082	76.0542	76.1002	76.1924
166	76.7462	76.8382	76.9312	76.9775	77.0238	77.1165
167	77.6737	77.7667	77.8598	77.9064	77.9530	78.0462
168	78.6067	78.7003	78.7940	78.8408	78.8877	78.9815
169	79.5452	79.6393	79.7335	79.7806	79.8278	79.9222
170	80.4894	80.5841	80.6789	80.7263	80.7737	80.8686
171	81.4391	81.5344	81.6297	81.6774	81.7251	81.8206
172	82.3944	82.4902	82.5861	82.6340	82.6820	82.7780
173	83.3553	83.4517	83.5481	83.5963	83.6446	83.7412
174	84.3217	84.4186	84.5156	84.5641	84.6127	84.7098
175	85.2937	85.3912	85.4888	85.5376	85.5864	85.6842

.5	.6	.7	.75	.8	.9
55.7640	55.8428	55.9217	55.9612	56.0007	56.0797
56.5549	56.6343	56.7138	56.7535	56.7933	56.8729
57.3515	57.4315	57.5115	57.5515	57.5915	57.6716
58.1536	58.2341	58.3147	58.3550	58.3953	58.4760
58.9613	59.0424	59.1235	59.1641	59.2047	59.2859
59.7745	59.8561	59.9378	59.9787	60.0195	60.1013
60.5933	60.6755	60.7577	60.7988	60.8400	60.9224
61.4177	61.5004	61.5832	61.6246	61.6661	61.7490
62.2477	62.3310	62.4144	62.4561	62.4978	62.5813
63.0832	63.1671	63.2510	63.2930	63.3350	63.4190
63.9243	64.0087	64.0932	64.1355	64.1777	64.2623
64.7710	64.8560	64.9410	64.9835	65.0261	65.1112
65.6232	65.7087	65.7943	65.8371	65.8799	65.9656
66.4810	66.5671	66.6532	66.6963	66.7394	66.8257
67.3444	67.4310	67.5177	67.5611	67.6045	67.6913
68.2134	68.3006	68.3878	68.4314	68.4751	68.5625
69.0879	69.1757	69.2635	69.3074	69.3514	69.4393
69.9680	70.0563	70.1447	70.1889	70.2331	70.3216
70.8536	70.9425	71.0314	71.0759	71.1204	71.2095
71.7449	71.8343	71.9238	71.9685	72.0133	72.1029
72.6417	72.7317	72.8217	72.8667	72.9118	73.0020
73.5440	73.6345	73.7251	73.7704	73.8158	73.9065
74.4520	74.5431	74.6342	74.6798	74.7254	74.8167
75.3655	75.4572	75.5489	75.5948	75.6407	75.7325
76.2846	76.3768	76.4691	76.5152	76.5614	76.6538
77.2092	77.3020	77.3948	77.4412	77.4877	77.5807
78.1395	78.2328	78.3262	78.3729	78.4196	78.5131
79.0753	79.1692	79.2631	79.3101	79.3571	79.4511
80.0166	80.1111	80.2056	80.2529	80.3002	80.3948
80.9636	81.0586	81.1536	81.2011	81.2487	81.3439
81.9161	82.0117	82.1073	82.1551	82.2029	82.2986
82.8741	82.9702	83.0664	83.1145	83.1626	83.2589
83.8378	83.9345	84.0312	84.0796	84.1280	84.2248
84.8070	84.9042	85.0015	85.0501	85.0988	85.1962
85.7818	85.8796	85.9774	86.0263	86.0753	86.1733

Di. Inches	0	.1	.2	.25	.3	.4
176	86.2713	86.3694	86.4675	86.5176	86.5656	86.6638
177	87.2544	87.3530	87.4517	87.5010	87.5504	87.6492
178	88.2431	88.3422	88.4414	88.4910	88.5407	88.6401
179	89.2374	89.3371	89.4369	89.4868	89.5367	89.6366
180	90.2372	90.3375	90.4379	90.4881	90.5383	90.6388
181	91.2427	91.3435	91.4444	91.4949	91.5454	91.6464
182	92.2536	92.3550	92.4565	92.5573	92.5580	92.6596
183	93.2702	93.3722	93.4742	93.5253	93.5763	93.6784
184	94.2923	94.3948	94.4974	94.5487	94.6000	94.7027
185	95.3200	95.4231	95.5263	95.5779	95.6295	95.7327
186	96.3533	96.4569	96.5606	96.6124	96.6643	96.7681
187	97.3920	97.4962	97.6005	97.6526	97.7048	97.8092
188	98.4366	98.5413	98.6461	98.6985	98.7510	98.8559
189	99.4866	99.5919	99.6972	99.7499	99.8026	99.9081
190	100.5421	100.6480	100.7539	100.8069	100.8599	100.9659
191	101.6032	101.7096	101.8161	101.8694	101.9226	102.0292
192	102.6699	102.7769	102.8840	102.9375	102.9911	103.0982
193	103.7422	103.8497	103.9573	104.0111	104.0649	104.1726
194	104.8200	104.9281	105.0362	105.0903	105.1444	105.2527
195	105.9034	106.0120	106.1207	106.1751	106.2295	106.3383
196	106.9924	107.1016	107.2109	107.2655	107.3202	107.4296
197	108.0869	108.1967	108.3065	108.3614	108.4164	108.5263
198	109.1871	109.2974	109.4078	109.4630	109.5182	109.6287
199	110.2927	110.4036	110.5145	110.5700	110.6255	110.7366
200	111.4040	111.5154	111.6269	111.6827	111.7385	111.8501
201	112.5208	112.6328	112.7448	112.8009	112.8569	112.9691
202	113.6432	113.7557	113.8683	113.9247	113.9810	114.0937
203	114.7712	114.8843	114.9975	115.0541	115.1107	115.2240
204	115.9047	116.0183	116.1320	116.1889	116.2458	116.3597
205	117.0438	117.1580	117.2723	117.3295	117.3866	117.5010
206	118.1885	118.3032	118.4180	118.4755	118.5329	118.6479
207	119.3388	119.4541	119.5695	119.6272	119.6849	119.8004
208	120.4946	120.6105	120.7264	120.7844	120.8424	120.9585
209	121.6560	121.7724	121.8889	121.9472	122.0054	122.1220
210	122.8229	122.9399	123.0570	23.1156	123.1741	123.2913

.5	.6	.7	.75	.8	.9
86.7621	86.8604	86.9588	87.0081	87.0573	87.1558
87.7481	87.8470	87.9460	87.9955	88.0450	88.1440
88.7396	88.8391	88.9386	88.9884	89.0382	89.1378
89.7366	89.8366	89.9367	89.9868	90.0368	90.1370
90.7393	90.8399	90.9405	90.9909	91.0412	91.1419
91.7475	91.8486	91.9498	92.0004	92.0510	92.1523
92.7612	92.8629	92.9646	93.0155	93.0664	93.1683
93.7806	93.8828	93.9851	94.0363	94.0874	94.1898
94.8055	94.9083	95.0112	95.0627	95.1141	95.2170
95.8360	95.9393	96.0427	96.0945	96.1462	96.2497
96.8720	96.9759	97.0799	97.1319	97.1839	97.2879
97.9137	98.0182	98.1227	98.1750	98.2273	98.3319
98.9609	99.0659	99.1710	99.2236	99.2761	99.3813
100.0136	100.1192	100.2248	100.2777	100.3305	100.4363
101.0720	101.1781	101.2843	101.3374	101.3905	101.4966
102.1359	102.2426	102.3493	102.4027	102.4561	102.5630
103.2054	103.3126	103.4199	103.4736	103.5273	103.6347
104.2804	104.3882	104.4961	104.5500	104.6040	104.7120
105.3610	105.4694	105.5778	105.6320	105.6863	105.7948
106.4472	106.5561	106.6651	106.7196	106.7741	106.8832
107.5390	107.6485	107.7580	107.8128	107.8676	107.977[illegible]
108.6363	108.7463	108.8564	108.9115	108.9666	109.0768
109.7392	109.8498	109.9604	110.0157	110.0711	110.1819
110.8477	110.9589	111.0701	111.1257	111.1813	111.2926
111.9617	112.0734	112.1852	112.2411	112.2970	112.4089
113.0813	113.1936	113.3059	113.3621	113.4183	113.5307
114.2065	114.3193	114.4322	114.4887	114.5451	114.6581
115.3373	115.4507	115.5641	115.6209	115.6776	115.7911
116.4736	116.5876	116.7016	116.7586	116.8156	116.9297
117.6155	117.7300	117.8446	117.9019	117.9592	118.0738
118.7629	118.8780	118.9931	119.0507	119.1083	119.2235
119.9160	120.0316	120.1473	120.2051	120.2630	120.3788
121.0746	121.1908	121.3070	121.3651	121.4233	121.5396
122.2387	122.3554	122.4722	122.5306	122.5890	122.7059
123.4085	123.5258	123.6431	123.7018	123.7605	123.8779

Di. Inches	0	.1	.2	.25	.3	.4
211	123.9954	124.1129	124.2305	124.2894	124.3482	124.4660
212	125.1735	125.2916	125.4098	125.4689	125.5280	125.6463
213	126.3572	126.4758	126.5945	26.6539	126.7133	126.8322
214	127.5464	127.6656	127.7849	127.8446	127.9042	128.0236
215	128.7412	128.8610	128.9809	129.0408	129.1008	129.2207
216	129.9416	130.0620	130.1824	130.2426	130.3028	130.4233

.5	.6	.7	.75	.8	.9
124.5838	124.7017	124.8196	124.8785	124.9375	125.0555
125.7647	125.8831	126.0016	126.0608	126.1201	126.2386
126.9511	127.0701	127.1891	127.2485	127.3081	127.4272
128.1431	128.2626	128.3822	128.4420	128.5018	128.6215
129.3407	129.4607	129.5808	129.6409	129.7010	129.8213
130.5439	130.6645	130.7852	130.8456	130.9059	131.0267

A CYCLOMETRICAL TABLE in Wine Gallons *and* Decimilleſſi-*Quarter of an Inch of the Diameter, according to* 231 *Cubical Inches to ſuch*

Circles Area's

Diâ. in Inch.	0	.1	.2	.25	3.	.4.
1	0.0034	0.0041	0.0049	0.0053	0.0057	0.0067
2	0.0136	0.0150	0.0165	0.0173	0.0180	0.0196
3	0.0306	0.0326	0.0347	0.0359	0.0369	0.0392
4	0.0544	0.0572	0.0600	0.0515	0.0629	0.0659
5	0.0580	0.0884	0.0919	0.0937	0.0955	0.0991
6	0.1224	0.1265	0.1307	0.1328	0.1350	0.1393
7	0.1666	0.1714	0.1763	0.1787	0.1812	0.1862
8	0.2176	0.2231	0.2287	0.2315	0.2343	0.2400
9	0.2754	0.2815	0.2877	0.2910	0.2944	0.3004
10	0.3400	0.3468	0.3537	0.3572	0.3607	0.3678
11	0.4114	0.4189	0.4265	0.4303	0.4341	0.4418
12	0.4896	0.4978	0.5061	0.5102	0.5144	0.5228
13	0.5746	0.5835	0.5925	0.5970	0.6015	0.6106
14	0.6654	0 6760	0.6856	0.6904	0.6953	0.7051
15	0.7650	0.7752	0.7855	0.7907	0.7959	0.8064
16	0 8704	0.8813	0.8923	0.8978	0.9034	0.9145
17	0.9826	0.9942	1.0059	1.0117	1.0176	1.0294
18	1.1016	1.1139	1.1263	1.1325	1.1386	1.1511
19	1.2274	1.2404	1.2534	1.2600	1.2665	1.2797
20	1.3600	1.3736	1.3873	1.3942	1.4011	1.4149
21	1.4994	1.5137	1.5281	1.5353	1.5425	1.5570
22	1.6456	1.6606	1.6757	1.6827	1.6908	1.7060
23	1.7986	1.8142	1.8299	1.8378	1.8457	1.8616
24	1.9584	1.9747	1.9911	1.9994	2.0076	2.0242
25	2.1250	2.1420	2.1591	2.1677	2.1763	2.1936

exhibiting the Area's of CIRCLES *from One Inch unto Five Feet Diameter, mal Parts, Calculated to each Tenth and Gallon.*

in Wine Gallons.

.5	.6	.7	.75	.8	.9
0.0077	0.0087	0.0098	0.0104	0.0110	0.0123
0.0213	0.0230	0.0248	0.0258	0.0267	0.0286
0.0416	0.0440	0.0465	0.0478	0.0491	0.0517
0.0689	0.0720	0.0751	0.0736	0.0783	0.0816
0.1029	0.1067	0.1106	0.1126	0.1145	0.1184
0.1437	0.1482	0.1527	0.1549	0.1573	0.1619
0.1913	0.1964	0.2016	0.2047	0.2079	0.2122
0.2457	0.2515	0.2574	0.2603	0.2633	0.2693
0.3069	0.3134	0.3200	0.3233	0.3266	0.3333
0.3749	0.3821	0.3893	0.3929	0.3966	0.4040
0.4497	0.4576	0.4655	0.4694	0.4735	0.4815
0.5313	0.5398	0.5484	0.5527	0.5571	0.5658
0.6197	0.6289	0.6382	0.6428	0.6475	0.6569
0.7149	0.7248	0.7348	0.7398	0.7448	0.7549
0.8169	0.8275	0.8381	0.8434	0.8488	0.8596
0.9257	0.9369	0.9482	0.9539	0.9596	0.9711
1.0413	1.0532	1.0652	1.0712	1.0773	1.0894
1.1637	1.1763	1.1890	1.1953	1.2017	1.2145
1.2929	1.3062	1.3195	1.3262	1.3329	1.3464
1.4289	1.4428	1.4569	1.4639	1.4710	1.4851
1.5716	1.5862	1.6009	1.6087	1.6157	1.6306
1.7213	1.7366	1.7520	1.7597	1.7675	1.7830
1.8777	1.8937	1.9098	1.9179	1.9259	1.9421
2.0409	2.0576	2.0744	2.0828	2.0912	2.1081
2.2109	2.2283	2.2458	2.2546	2.2633	2.2808

Dia. in Inch.	0	.1	.2	.25	.3	.4
26	2.2984	2.3161	2.3336	2.3425	2.3513	2.3697
27	2.4786	2.4970	2.5155	2.5247	2.5340	2.5526
28	2.6656	2.6847	2.7038	2.7134	2.7230	2.7423
29	2.8594	2.8791	2.8989	2.9089	2.9188	2.9388
30	3.0600	3.0804	3.1009	3.1112	3.1215	3.1422
31	3.2674	3.2885	3.3097	3.3203	3.3309	3.3522
32	3.4816	3.5034	3.5253	3.5362	3.5472	3.5692
33	3.7026	3.7251	3.7476	3.7589	3.7702	3.7929
34	3.9304	3.9536	3.9768	3.9885	4.0001	4.0235
35	4.1650	4.1888	4.2127	4.2247	4.2367	4.2608
36	4.4064	4.4309	4.4555	4.4679	4.4802	4.5049
37	4.6546	4.6798	4.7051	4.7178	4.7304	4.7558
38	4.9096	4.9355	4.9614	4.9744	4.9874	5.0135
39	5.1714	5.1980	5.2246	5.2380	5.2513	5.2781
40	5.4400	5.4673	5.4946	5.5083	5.5220	5.5494
41	5.7154	5.7434	5.7714	5.7854	5.7994	5.8275
42	5.9986	6.0262	6.0549	6.0692	6.0836	6.1124
43	6.2866	6.3159	6.3452	6.3599	6.3746	6.4041
44	6.5824	6.6124	6.6424	6.6574	6.6725	6.7027
45	6.8850	6.9156	6.9463	6.9617	6.9771	7.0079
46	7.1944	7.2257	7.2571	7.2728	7.2886	7.3201
47	7.5106	7.5426	7.5747	7.5908	7.6068	7.6390
48	7.8336	7.8663	7.8990	7.9154	7.9318	7.9647
49	8.1634	8.1968	8.2302	8.2469	8.2637	8.2973
50	8.5000	8.5340	8.5681	8.5852	8.6023	8.6366
51	8.8434	8.8781	8.9129	8.9304	8.9478	8.9827
52	9.1936	9.2290	9.2645	9.2823	9.3000	9.3356
53	9.5506	9.5867	9.6228	9.6409	9.6590	9.6953
54	9.9144	9.9512	9.9880	10.0065	10.0249	10.0619
55	10.2850	10.3224	10.3599	10.3787	10.3975	10.4352
56	10.6624	10.7005	10.7387	10.7579	10.7770	10.8153
57	11.0466	11.0854	11.1243	11.1437	11.1632	11.2023
58	11.4376	11.4771	11.5166	11.5364	11.5562	11.5959
59	11.8354	11.8755	11.9157	11.9359	11.9560	11.9964
60	12.2400	12.2808	12.3217	12.3422	12.3627	12.4038

.5	.6	.7	.75	.8	.9
2.3877	2.4057	2.4238	2.4329	2.4419	2.4603
2.5713	2.5900	2.6088	2.6182	2.6277	2.6466
2.7617	2.7811	2.8006	2.8104	2.8201	2.8397
2.9589	2.9789	2.9992	3.0093	3.0194	3.0396
3.1629	3.1837	3.2045	3.2149	3.2254	3.2464
3.3736	3.3951	3.4166	3.4274	3.4382	3.4599
3.5913	3.6134	3.6356	3.6467	3.6578	3.6802
3.8157	3.8385	3.8614	3.8729	3.8843	3.9073
4.0469	4.0704	4.0940	4.1058	4.1176	4.1413
4.2849	4.3091	4.3333	4.3455	4.3576	4.3820
4.5297	4.5545	4.5794	4.5919	4.6044	4.6295
4.7813	4.8068	4.8324	4.8552	4.8581	4.8838
5.0397	5.0659	5.0922	5.1054	5.1185	5.1449
5.3049	5.3318	5.3588	5.3723	5.3858	5.4128
5.5769	5.6044	5.6320	5.6458	5.6597	5.6875
5.8557	5.8840	5.9123	5.9265	5.9407	5.9691
6.1413	6.1702	6.1992	6.2138	6.2283	6.2574
6.4337	6.4633	6.4930	6.5079	6.5227	6.5525
6.7329	6.7632	6.7935	6.8087	6.8239	6.8544
7.0389	7.0699	7.1010	7.1165	7.1321	7.1632
7.3517	7.3834	7.4151	7.4310	7.4469	7.4787
7.6713	7.7036	7.7360	7.7522	7.7685	7.8010
7.9977	8.0307	8.0638	8.0804	8.0969	8.1301
8.3309	8.3649	8.3984	8.4153	8.4322	8.4661
8.6709	8.7053	8.7397	8.7570	8.7742	8.8088
9.0177	9.0528	9.0879	9.1055	9.1231	9.1583
9.3713	9.4070	9.4428	9.4607	9.4787	9.5146
9.7317	9.7681	9.8046	9.8229	9.8411	9.8777
10.0989	10.1360	10.1731	10.1917	10.2103	10.2476
10.4729	10.5107	10.5485	10.5674	10.5864	10.6244
10.8537	10.8922	10.9307	10.9500	10.9693	11.0079
11.2413	11.2804	11.3196	11.3393	11.3589	11.3982
11.6357	11.6755	11.7154	11.7354	11.7553	11.7953
12.0369	12.0774	12.1180	12.1383	12.1586	12.1993
12.4449	12.4861	12.5273	12.5480	12.5686	12.6100

XXIV. Probl.

Fig. 7. *Having* a C b, *the Axe of a Globe or Sphear,* 200 *Inches*; *To find its Capacity in Wine Gallons.*

The Rule.

Multiply the Cube of the Axe (if the Capacity be inquired in Wine Gallons) by .002266661—; and if in Ale Gallons, multiply the Cube of the Axe alwayes by .001856733—, and that Product is your desire.

To find the Solidity in Cubick Inches, mult. the Cube of the Axe by .523598775598 3-+ the $\frac{1}{6}$ of the Area of that Circle, whose Diameter is 2, and that Product is your desire.

200
200
40000
200
8000000

Multiplicator = .002266661—
Cube of Sphears Axe = 8000000
Respons. Wine Gallons 18133.288000000

Another

Another way, per *Cyclometrical Table.*

$\frac{2}{3}$ of Area Circle, whoſe Diameter = 200 Inch.
per Table = Wine Gall. 90.66645443-|-
The Sphears Axe = 200
Reſponſ. 18133.29088600
prout ſupra.

The Converſe.

Having the liquid Capacity of a Globe; To find the quantity of its Axe in Inches.

The Rule.

Multiply the Globes propoſed Capacity, in Wine Gallons, by 441.1775-|-, and that Product is equal to the Cube of the Globes Axe ſought, whereof the Cubick Root is the Axe it ſelf; but if the Capacity be propoſed in Ale Gallons, the Multiplicator is = 538.58033-|-

As for INSTANCE.

There is a Globe, whoſe whole Capacity is 18.133291 *Wine Gallons, how many Inches are this Globes Axe?*

18.133291+
441.1775
90666455
126933037
126933037
18133291
18133291
72533164
72533164

7999.9999901525 = 8000, whereof the Cubick Root, *viz.* 20 Inch. = the Globes Axe.

XXV. Probl.

Fig. 11. *Having the Diameter at the Base of the frustum, or portion of a Globe (intercepted between two Planes parallel, one touching, and the other cutting the Globe) together with the Frustum's altitude; To find the Globes Axe.*

The Rule.

The Square of the Semidiameter of the Frustums Base, divided by the greater or lesser Frustums altitude, or multiplied by its proper Multiplicator, gives the Remainder of the Globes Axe, which added unto the greater or lesser Frustums altitude, inquired of, that Sum is equal to the Globes Axe sought.

As

As for Example.

Let *c o d* the Diameter of the Fruſtums Baſe be 16 Inches,
And *o x* its altitude 4
And *e m* the Axe ſought, —
q. of Semidiam. of Fruſtums Baſe = 64
The Multiplicator for 4 altitude = .25
320
128
The greater Fruſtums altitude = 16.
The given Fruſtums altitude = 4
Reſponſ. the Globes Axe = 20 Inches.

XXVI. PROBL.

Having the Altitude of the Fruſtum, or portion of a Globe, and the Globes Axe: To find the Diameter of the Fruſtums Baſe. Fig. 11.

As for INSTANCE.

The Axe of a Globe is 20 *Inches, and the Altitude of its Fruſtum* = 4 *Inches; how many Inches make the Diameter of the Fruſtums Baſe?*

The Rule.

A Geometrical Mean proportional between the greater and leſſer Fruſtums Altitudes, is equal

equal to the Semidiameter of the Fruſtums Baſe, which doubled, is the Diameter of the Baſe required.

The Fruſtums Altitude propoſed = 4 Inches
And conſequently the greater Fr. Alt. $\overline{16}$
$\overline{64}$
Whereof the Square Root = 8
$\overline{2}$
Reſponſ. The Diameter of the Baſe $\overline{16}$

XXVII. PROBL.

Fig. 11. *Having the Axe of a Globe and the Altitude of its Fruſtum; To find the Fruſtums liquid Capacity, without knowledge of the Diameter of the Baſe.*

Let the Globes Axe be 20 *Inches, and the Altitude* 4, *and the Fruſtums Capacity be required in Wine Gallons.*

The Rule.

(1.) From the Globles Axe ſubduct the Fruſtums Altitude, and unto that Remainder add the Globes Semi-axe. (2.) Multiply that Summ by the Square of the Fruſtums Altitude; which done, that Product being multiplied (for Wine Gallons), by .00453323+; for Wine Pints, by .03626658+; and for Ale Gallons, by .00371346 7+ produces the Fruſtums Capacity required.

The

	Inches.
The Globes Axe =	20
The Altitude $x\ o$ =	4
The greater Fruſtums Altitude =	16
The Globes Semi-Axe =	10
	26
The Square of the Altitude given =	16
	156
	26
	416

The Mult. for Wine Gall. = .004533323 +

416

27199938

4533323

18133292

Reſponſ. in Wine Gallons 1.885862368

XXVIII. Probl.

Having the Altitude of a Globes Fruſtum, with the Diameter of its Baſe; To find the Fruſtums liquid Capacity, without knowledge of the Globes Axe. Fig. 11.

The

The Rule

(1.) Square *c d*, the Diameter of the Fruſtums Baſe, and that Square multiply by the Altitude, *x o*, reſerving the Product.

(2.) Cube *x o*, the Fruſtums Altitude, and thereto add the $\frac{1}{3}$ of that Cube, and this Summ add unto the before reſerved Product; which done, that Aggregate mult. by .0016999997+ for Wine Gallons, or by .01359998+ for Wine Pints, or by .00139255 for Ale Gallons, produces the Queſtions Anſwer.

As for INSTANCE.

Let the Altitude o x *be* 4 *Inches, and the Diameter of the Baſe* 16, *and the Capacity be inquired in Wine Gallons.*

$$\begin{array}{rl} c\,d = & 16 \\ & 16 \\ \hline & 96 \\ & 16 \\ \hline & 256 \\ \text{Altitude} = & 4 \\ \hline & 1024 \end{array}$$

Cube

Cube of x 0 4 = 64
Its $\frac{1}{3}$ = 21.3333
1024

1109.3333
Multiplicator for Wine Gallons = .0017+

77653331
11093333

Respons. Wine Gallons 1.88586661
The Fruſtums Capacity.

Another way, the Rule is.

(1.) Square the Diameter of the Fruſtums Baſe. (2.) Square the Fruſtums Altitude, and thereof take the $\frac{1}{3}$; which done, add theſe three Numbers together, and their Summ multiply by the Fruſtums Altitude; laſtly, that Product multiplied by any of the above given Multiplicators, according to the purport of the Queſtion gives your deſire.

q. of Diameter at the Baſe 16 = 256
q. of 0 x the Fruſtums Altitude = 16
Its $\frac{1}{3}$ = 5.3333

277.3333
0 x the Fruſtums Altitude = 4

1109.3332

1109.3332
Multiplicator for Wine Gallons = .0017+

77653331
11093333

Respons. Wine Gallons 1.88586661
prout supra.

XXIX. PROBL.

Fig. 11. *Having the Axe of a Globe or Sphere,* e m, 20 *Inches, and* z æ 6, *the Altitude of the Zone of the Globe,* r æ S m z e, *together with the Diameter of its lesser Base* r S, 16 *Inches; To find the Zones Capacity* r æ S m z e *in Wine Gallons.*

The Rule.

(1.) Find $\frac{2}{3}$ of the Area of the greater Base, by the Cyclometrical Table, and then the $\frac{1}{3}$ of the Area of the lesser Base, either in Wine or Ale Gallons, according as the Question shall require. (2.) Having added them together, multiply their Summ by the Zones Altitude; (which done) that Product is your desire.

$\frac{2}{3}$ of

$\frac{2}{3}$ of Area of that Circ. whose Diameter is = *e z m*, the greater Base, 20 Inches,	*per* T. = .9067 of a W. Gal.
$\frac{1}{3}$ of Area of that Circ. whose Diameter is = *r æ S* the lesser Base 16 Inches,	*per* T. = .2901
Their Summ	1.1968
Zones Altitude =	6
The Zones whole Capacity =	7.1808

Or more briefly, work thus, viz.

	Wine Gall.
Of the Axe 20 Inch. the Area *per* Table doubled	= 2.7200
Of the lesser Bases Diameter (16) the Area *per* Table	= 0.8704
Their Summ	= 3.5904
Mult. *per* $\frac{1}{3}$ of the Zones Altitude, 6 =	2
Resp. the Zones Capacity (as before) =	7.1808

XXX. Probl.

There is a Globe of Glass, whereof the liquid Capacity is 4 Wine Gallons, the Proprietor bespeaks of a Workman a Cubical Vessel (i. e.) *a Vessel, whose length, breadth, and depth are equal) that shall be equal in Capacity unto this Globe; The Question is, how many Inches the side of the*

the Cube must be, as also, how many Inches the Globes Axe is of?

The Rule.

(1.) The Cubick Root of the Cubical Inches in the Globes Solidity, is equal to the side of the Cube, that is, equal in Solidity thereunto. (2.) The Globes Capacity in Wine Gallons; multiplied by 441.1775, produces the Cube of the Globes Axe sought.

Cub. Inch. in a W. Gall. 231
Globes Capacity = 4 Gallons.

2,9656720 : 924
0,9885573 *Resp.* 9.739961 Inches, the side of the Cube sought.

441.1775
4

3,2466733 : 1764.71 ..
1,0822244 *Responſ.* = 12.08439 Inches, the Globes Axe inquired of.

XXXI. PROBL.

There is a Cylinder, whereof the Diameter at the Baſe is 144 *Inches, and the Altitude* 240; *how many Ale Gallons doth each wet Inch of it contain, and how many are its whole Capacity?* Fig. 8.

The Rule.

Multiply the Square of the Diameter at the Baſe by the proper Multiplicator (being the ſame with thoſe given for a whole Cone *Probl.* 35) and that product ſhews each wet Inches Capacity; which multiplied by the whole Altitude, or by any propoſed Altitude of the Liquor, gives the whole Capacity of the Cylinder, or ſhews what Liquor remains in it, at any Altitude of Liquor inquired of.

144
144

576
576
144

20736

The proper Multiplicator for Ale Gallons
= .0027851+

The Square of Diameter of the Base 20736

167106
83553
194957
557020

Respons. Ale Gall. *per* wet Inch 57.751 8336
The Altitude = 240

23100733440
1155036672

And 13860,4400640

The whole Capacity, being exactly triple to the Cones Capacity of the same Base and Altitude.

Another way per *Cyclometrical Table.*

144 Inches Diam. *per* Tab. } Ale Gallons.
each Inch. Capac. given *per* } = 57.751831+
Tab. by ocular inspection, } Altitude = 240

2310073240
115503662

Respons. prout suprà 13860.439440

Note, in dry Measure, That 272 Cubical Inches are in the Gallon for Corn, Coles, Salt, *&c.* two Gallons make a Peck.

XXXII.

XXXII. Probl.

Some Uſes and Improvements of the precedent TABLE.

A Brewer beſpeaks of a Back maker a Cylindrical Tunn, i. e. *of equal Diameters from top to bottom and of ſuch a magnitude, as to hold 6 Gallons, Beer meaſure, upon every Inch; the Queſtion is, how many Inches this Tuns Diameter muſt be?*

The Analogiſm.

As the $\frac{1}{3}$ part of a Gallon, is to the Square of 18.9487 Inches, *(i. e.)* the Gauge Point in Beer meaſure, being the Diameter of that Circle, whoſe Area = 282 Square Inches) ſo is the $\frac{1}{3}$ of the Area of the Circle inquired of, to wit, (12 Gallons) unto the Square of the Diameter, whereof the Square Root reſolves the Queſtion.

But by way of Contraction.

A general Multiplicator 1077.161
$\frac{1}{3}$ of Area propoſed = 12 Gallons

12925.932

Whoſe Square Root being 113.694 Inches, is the Diameter required.

But this Table reſolves all ſuch like Queſtions by ocular inſpection only; it appears by the Table, that if the Diameter be made 113.7, it will hold upon every Inch 36.0049 Gallons: In like manner alſo, if the Tunn had been to hold 35 $\frac{1}{2}$ Gall. upon every Inch, it appears by the Table, that its Diameter then muſt have been but 112.$\frac{2}{10}$ Inches: And in caſe the Tunn were to hold 18 Gallons upon each wet Inch, the Diameter then ſhould have been 80.4 very nigh.

There is a Cylindrical Tunn or Veſſel, whoſe whole Capacity is 49 *Beer Barrels, and* 11 *Gall. and depth* 50 *Inches; what is the Diameter of this Tunns Baſe?*

The Tunns Capacity in Gall. = 1775
Multiplicator for the depth 50 = .02

The Tunn holds *per* Inch = 35.5 Gallons.
Againſt 35.5 Gal. I find *per* Tab. = *Reſp.* 112.$\frac{2}{10}$ Inches, the Diameter ſought.

Fig. 9. *There is a Conical Tunn, whereof the upper Diameter* b,a, *is* 144 *Inches, and lower Diameter* d,c, 108; *and the depth,* 60 *Inches: how may Beer Gallons is this Tunns liquid Capacity?*

b a = 144
d c = 108
—— leſſer depth
As dif. of Diam. 36 to 108 :: 60 : 180
180

180 = *z*, *o*, Alt. of the Cones Fruſtum cut off.
60 = *n*, *o*, the depth.

240 = *z*, *n*, the Alt. or Axe of whole Cone.

Of 144 Inches Diameter, the Area *per* Table
= 57.7518 Beer Gall.
The $\frac{1}{3}$ of Cones Axe = 80

Whole Cones Capac. = 4620.1440

Of 108 Inches Diameter, the Area per Table
= 32.4854
The $\frac{1}{3}$ of *z*, *o*, = 60

1949.1240 the Capacity of *c*, *d*, *z*, the Cone cut off.

Reſp. 2671.020 Ale Gallons, the Tunns Capacity required. And thus with much facility may the Tunn be Inched by aid of the Table. But of this, hereafter.

XXXIII. Probl.

A Cylinders liquid Capacity is 1775 *Ale Gallons, the Diameter of its Baſe* 112.9 *Inch. what is this Cylinders Altitude?*

The Rule.

By the Cyclometrical Table it doth upon ocular inſpection appear that a Cylinder whoſe

Diameter is 112.9 Inches, doth hold upon each wet Inch 35.5 Ale Gallons; which is equal to the Area of a Circle of that Diameter: And then the given Capacity being divided by the Circles Area (given *per* Table) Quotes, or being multiplied by the proper Multiplicator for the Circles Area (found by Rule in *Pr.* 59.) produces the Cylinders Altitude sought.

The proper Multiplicator for the Circles Area
35.5 Gallons = .0281691+
Cylinders liquid Capacity = 1775 Ale Gall.

1408455
1971837
1971837
281691

50(0001525

Respons. The Altitude = 50 Inches.

XXXIV. Probl.

There is a Tunn or Vessel in the form of a Cylinder, cut with a Plane, parallel to the Axe, the versed Sine or Arrow = 111 Inches, and the whole Diameter, 400 Inches; and the depth, 58 Inches; How many Ale Gallons may this whole Vessel contain, and how much per *each wet Inch?*

Area

Area of the Cylinder = 445.616 Ale Gallons,
Depth = 58

25845.728 Capacity of the whole Cylinder.

By the Table of Area's of Segments, thus,

Multiplicator for Diameter 400 = .0025
Arrow = 111

25
Com.diff. 25
.0010 : 114 : .0005 : .00057 25

.2775

.2770

Difference .0005

I find .2770 *per* Table = .22578
...57

So that .2775 = .22635
Mult. *per* Capac. of whole Cylin. = 25845.728

Resp. Veſſ. whole Capac. = Ale Gall. 5850.1805

Multipl. for depth 58 Inch. = 01724138
Multiply *per* Veſſ. Capacity = 5850.1805

Resp. Veſſel holds *per* wet Inch 100.8652, *&c.* Ale Gallons.

XXXV.

XXXV. Probl.

Fig. 9. *There is a Cone, as* a, b, z, *whereof the Altitude, or Axe* z, n, *is* 240 *Inches, and* a, b, *the Diameter of its Base* = 144 *Inches: How many Ale Gallons is this Cones Capacity?*

The Rule.

Mult. the q. of the Diam. of the Base by $\frac{1}{3}$ of the Cones Altitude, and that Product multiplied by the proper Multiplicator given, produces the Cones Capacity sought.

The proper Multiplicator in a whole Cone or Cylinder for

Wine Gallons	.00339992—
Wine Pints	.02719993—
Ale Gallons	.0027851+
Ale Pints	.0222808+
Ale Barrels	.00008703437+
Beer Barrels	.00007736 39—

144
144
576
576
144
20736

The

The Square of Diameter at the Base 20736
The $\frac{1}{3}$ of the Cones Altitude 240 = 80

1658880
The Multiplicator for Ale Gallons .0027851

1658880
8294400
13271040
11612160
3317760

Resp. Ale Gallons 4620.1466880

Another way, per *Cyclometrical Table.*

Ale Gallons.
144 Inches Diam. *per* Table = 57.751831+
$\frac{1}{3}$ of the Cones Altitude = 80

Respons. 4620.146480
prout supra.

Being exactly $\frac{1}{3}$ of the Capacity of a Cylinder of the same Base and Altitude.

XXXVI.

XXXVI. Probl.

Fig. 9. *Having the Dimensions of the Conical Tunn* a, b, d, c, viz. *the greater Diameter* a, b, = 144 *Inches, the lesser Diameter* c, d, = 108 *Inches, and the Tunns depth* n o = 60 *Inches; To find the Quantity of* z, n, *the whole Cones Axe.*

The Analogism.

As the difference of the Diameters, is unto the lesser of them; so is the Tunns depth, to the Altitude of the Cone, cut off, *z o*, whereunto the Tunns depth being added, that Summ is always = the Altitude or Axe of the whole Cone *z, n.*

144
108

36 : 108 :: 60 : 180 for *z, o.*

60 *n, o,* the depth.

240 = *z, n,* the whole Cones Axe, or Altitude.

XXXVII.

XXXVII. Probl.

There is a Conical Tunn, whereof the upper Diameter a b = 144 *Inches the leſſer Diameter* c, d, = 108 *Inches, and the Depth,* 60 *Inches*; = n, o : *How many Ale Gallons may this Tunn contain?* Fig. 9.

The Rule.

(1.) Square the Summ of the Diameters, and multiply the Diameters by one another, then deduct the later Product out of the former.

(2.) Multiply that difference (if the Capacity be deſired in ALE GALLONS) by .000928366+(if in BEER BARRELS) by .000025787+(if in ALE BARRELS)by .000029011+6,and(if in WINE GALLONS) by .001133331+ which done, that Product multiplied by the Tunns depth, produces the Capacity ſought.

144		144
108		108
252		1152
252		1440
504	The Rectangle of Diam. =	15552
1260		
504		
63504	The Square of Summ Diameters.	

63504 The Square of Summ Diameters.

15552 The Rectangle of Diameters.

47952 q. of Summ Diam. — their Rectangle.

Multiplicat. for Ale Gall. = .000928366+

47952

18567332
46418330
83552994
64985662
37134664

44.5170352(032

The Tunns Depth 60

Resp. Ale Gallons 2671.0221120 the Tunns Capacity.

Per *Cyclometrical Table, thus:*

Having found z, o, the Axe of the Cone, cut off; and z, n, the whole Cones Axe; as above.

Inch.

Of 144 Diameter the Area *per* T. = 57.7518

$\frac{1}{3}$ of z, n, 240 = 80

The whole Cones Capac. Ale Gall. 4620.1440

Of

Inch.

Of 108 Diameter the Area *per* T. = 32.4854

$\frac{1}{3}$ of $z,o,180 = 60$

The Capacity of the Cone cut off, *viz.* of c,d,z, = 1949.1240

Resp. The Tunns Capacity 2671.0200 *proximè, prout suprà.*

XXXVIII. Probl.

The Inching of a Conical Tunn in Ale Gallons, from the leſſer Diameter, upwards, by Multiplication and Addition only. Fig. 9.

The Rule.

1. As the Depth of the Tunn is to the Difference of its Diameters (taken in Inches) ſo is Unity, to the Decrement, or Increment of any two Diameters at 1 Inch diſtance, either upwards or downwards, which reſerve for a common Addend unto the increaſing Diameters.

2. Find the tripled Square of the Tunns leſſer Diameter, then find the tripled Rectangle of that Diameter and common Addend, laſtly ſquare that Addend: (which done) the Summ of thoſe three Numbers, being multiplied by one of the following Multiplicators (here ſubjoined) produceth the Tunns Capacity upon the firſt Inch, from the leſſer Diameter, in Ale Gallons, or Beer, or Ale Barrels.

Mul-

Multiplicator for { Ale Gallons .0009283666+ / Beer Barrels .0000257879+ / Ale Barrels .0000290114+ }

3. Set down again the tripled q. of the Diameter, and under it the nonocupled Rectang of that Diameter and Addend; and under them the ſeptupled Square of the Addend; (which done) the Summ of thoſe three numbers being multiplied by the ſaid proper Multiplicator (afore uſed) that Product ſhews the Tunns Capacity upon the next following Inch, (here the ſecond) from the leſſer Diameter; the Difference of which Capacities being always equal to the foremoſt of the ſecond Differences

4. The Summ of the ſextupled Rectangle of the leſſer Diameter and common Addend, and of the dodecupled Square of the ſame Addend, being multiplied by the proper Multiplicator, produces the ſecond of the ſecond Diff. *viz.* the Difference between the Tunns Capacity upon the Inch laſt found, and that of the following Inch (*viz.* here, between the ſecond and third Inch).

5. The ſextupled Square of the common Addend, multiplied by the aforeſaid Multiplicator, produces the third Difference, which varies not, but holds the ſame, throughout the work.

Then inſert to the Table the Tunns Capacity upon the firſt Inch, (being here found) 32.6662127, and under it the Capacity upon the ſecond Inch. *Example.*

Example.

Of the Conical Tunn *a, b, d, c,* let *a, b,* the greater Diameter be 144 Inches, *c, d,* the leſſer Diameter 108 Inches, and *n, o,* the Depth = 60 Inches. Fig. 9.

.01666667 Multiplicator for 60
36 Difference of Diameters.

10000002
5000001

.60000012 or $\frac{6}{10}$ Addend.

Leſſer Diameter = 108
The common Addend = .6

64.8
3
194.4

108
108
864
1080
11664
3
34992

34992

34992
194.4
The Square of Addend .36

35186.76

35186.76
.000928366+

211112056
211112056
211112056
10556028
28149408
7037352
31668084

Ale Gallons 32.666212746216

For what the Tunn holds upon the first Inch, from the lesser Diameter.

The tripled Square of the Diameter 108 = 34992
The nonocupled Rectangle of Diameter and Addend = 583.2
The septupled Square of common Addend = 2.52

35577.72

35577.72

35577.72
.00092836664

21346632
21346632
21346632
10673316
28462176
7115544
32019948

Ale Gallons 33.029166952152

The Tunn holds upon the second Inch from the leſſer Diameter.

The ſextupled Rectangle of the leſſer Diameter and Addend } = 388.8
The dodecupled Square of the Addend = 4.32

393.12

.00092836664
393.12

18567332
92836664
27850998
83552994
27850998

.364959477792

The ſecond of the ſecond Differences.

Multiplicator for Ale Gall. .000928366+
The sextupled Square of the Addend = 2.16

55701996
9283666
18567332

The third difference .002005271856
and this holds the same throughout the work.

Dept. in Inch.	*Tunns capacity upon each Inch in Ale Gallons.*	*Dept. in Inch.*	*Tunns capacity upon each Inch in Ale Gallons.*	*Dept. in Inch.*	*Tunns capacity upon each Inch in Ale Gallons.*
60	57.5116	40	48.3074	20	39.9052
59	57.0323	39	47.8682	19	39.5062
58	56.5551	38	47.4310	18	39.1092
57	55.0798	37	46.9959	17	38.7141
56	55.6066	36	46.5628	16	38.3211
55	55.1353	35	46.1316	15	37.9301
54	54.6661	34	45.7025	14	37.5410
53	54.1989	33	45.2754	13	37.1540
52	53.7336	32	44.8503	12	36.7690
51	53.2704	31	44.4271	11	36.3860
50	52.8092	30	44.0060	10	36.0050
49	52.3500	29	43.5869	09	35.6260
48	51.8928	28	43.1698	08	35.2490
47	51.4376	27	42.7547	07	34.8740
46	50.9844	26	42.3417	06	34.5010
45	50.5332	25	41.9306	05	34.1301
44	50.0849	24	41.5215	04	33.7611
43	49.6369	23	41.1144	03	33.3941
42	49.1917	22	40.7094	02	33.0292
41	48.7485	21	40.3063	01	32.6662

Ale Gallons = 2671.0231 the Tunns whole Capacity.

XXXIX. Probl.

The Inching of a Conical Tunn from the greater Diam. by Multiplication and Subduction only, in Ale Gallons, without the Cyclometrical Table.

The Rule.

1. As the Tunns Depth, is to the Difference of its Diameters; (taken in Inches) so is Unity, to the Increment or Decrement of any two Diameters at one Inch distance, either upwards, or downwards, the which reserve for a common Subducend, or Addend, according to which of the Diameters you begin at, whether the greater or lesser.

2. From the tripled Square of the greater Diameter subduct the tripled Rectangle of the greater Diameter and Subducend, and unto that Remainder add the Square of the Subducend; (which done) that Summ Multiplied by .000928366-+- for ALE GALLONS, by .000025787 97-+- for Beer Barrels, or by .000029011 46-+- for Ale Barrels, produceth the Tunns Capacity upon the first Inch from the greater Diameter, in Ale Gallons, or Barrels, according to the nature of the Multiplicator made use of.

3. From the tripled Square of the greater Diameter subduct the nonocupled Rectangle of

the greater Diameter and Subducend, and unto that Remainder add the ſeptupled Square of the Subducend; which done, that Summ multiplied by the aforeſaid Multiplicator, produces the Tunns Capacity upon the next Inch following, here the ſecond, from the greater Diameter.

4. From the ſextupled Rectangle of the greater Diameter and Subducend, ſubduct the dodecupled Square of the Subducend; which done, that Remainder multiplied by the ſaid Multiplicator, produces the ſecond Difference, *viz.* the Difference between the Tunns Capacity upon the Inch (laſt found) and that of the following Inch, here between the ſecond and third Inch.

5. The common Multiplicator aforeſaid, being multiplied by the ſextupled Square of the Subducend, produces the third Difference, which is common throughout the work.

Example.

Fig. 9. Of the Conical Tunn *a*, *b*, *d*, *c*, let *a*, *b*, (the gr. Diam.) be 144 Inches, *c*, *d*, (the leſſer Diam.) 108 Inches, and *n*, *o*, the depth = 60 Inches.

The Multiplicator for 60 = .01666667—
Difference of the Tunns Diameter = 36

10000002
5000001

.60000012, or $\frac{6}{10}$ the common Subducend or Addend through the work.

Diameter

Diameter *a*, *b*, = 144
Subducend = .6

86.4
3

259.2

a, *b*, 144
144

576
576
144

20736
3

Tripled Square of *a*, *b*, = 62208

Tripled Rectang. Diam. & Subduc. 259.2

61948.8
The Square of the Subducend = .36

61949.16
Multiplicator for Ale Gall. .0009283666 +

Ale Gallons = 57.5115310 &c.

The Tunns Capacity upon the first Inch from the greater Diameter.

Tripled Square of the greater Diam. 62208

9cupled Rectang. of Diam. & Subduc. 777.6

61430.4

Septupled Square of the Subducend = 2.52

61432.92

.000928366

57.0322711

&c.

The Tunns Capacity upon the second Inch from the greater Diameter.

57.5115310—

57.0322711—

.4792599

Sextupled Rectang. of Diam. & Subduc. 518.4

The dodecupled Square of Subducend = 4.32

514.08

.000928366+

.47725470 *&c.*

The second of the second Differences.

The Multiplicator = .000928366+

Sextupled q. of the Subducend 2.16

The third Difference = .002005271856

common throughout the work.

XI.

XL. Probl.

To find the Dripp, or Fall of any Tunn whatsoever.

The Rule.

Into the Tunn propoſed, (being empty) poure liquor, until the uppermoſt part, or edge of the bottom be juſt cover'd; then with a Gauging Rod (divided into Inches and Decimal Parts) take that liquors depth in the deepeſt place of it; laſtly, the one half of that depth (ſo taken) is *prope verùm* = the Tunns Fall or Dripp; (being always more, or leſs, according to the Tunns largeneſs or ſmallneſs;) the which being ſubducted from the Inches, *&c.* of the Tunns Depth (found by the Rod) leaveth the Veſſels true Depth, to be reckon'd on, and made uſe of in Inching that Tunn.

As for INSTANCE.

Admit there be a Conical Veſſel, whereof the upper Diameter is 216 *Inches, the lower* = 212, *and the Depth (found by the Rodd)* 12 *Inches: How many Inches of this Veſſels Depth are to be allowed for the Dripp or Fall; and what is the Veſſels true Depth, to be made uſe of in Inching the ſame?*

Finding the greateſt Depth of the liquor poured into this Veſſel (according to the ſaid Rule) to be 4 Inches; I conclude the true Fall of the Veſſel = 2 Inches; which taken out of the Veſſels whole Depth, (found by the Rod) 12 Inches; the Remainder, to wit, 10 Inches, is to be accounted the Veſſels true Depth, and to be made uſe of in Inching.

	Ale Gall.
The leſſer Diam. 214 Inch *per* T. =	127.5464
The depth from 2 Inches below the top, to 2 Inches above the bottom } =	8
	1020.3712
In the upper 2 Inches ———	127.5464
And in the lower two Inch. as much,	127.5464
	1275.4640

So this Tunns whole Capacity at 12 Inches deep, is directly equal to a Veſſel of thoſe Dimenſions that hath no Fall, and is but 10 Inches deep.

There being abated for this Tunns Fall 2 Inches of its Depth; as alſo, what it might have held upon thoſe 2 Inches.

See the Veſſel Inched in the Eighth Example of the following Problem.

XLI. Probl.

To Gauge and Inch a Conical, or Round Tunn, by aid of a ſliding Ruler, and the Cyclometrical Table.

The Rule

In Gauging of Round Tunns in general (or ſuch as to the eye ſeem ſo) obſerve theſe ſix Directions or Notes, *viz.*

1. *Direct.* Take with the ſliding Ruler the extream Diameters, *viz.* the Diameters at the top and bottom; then alſo take the ſame croſs-wiſe, as alſo in the midſt of the Tunns Depth, and likewiſe in the midſt of every Foot or half Foot of the Depth; noting well, whether the croſs Diameters do agree with each other, or not; for, if any difference of the extream croſs Diameters doth appear, both above and below; (as in the Ninth *Example*, or 11 *Probl.*) the Veſſel then is Elliptical, and to be Gauged, as there is directed: But if difference do ariſe only in one of them, to wit, in the croſs Diameters taken above; (as in the Tenth *Example* and 13 *Probl.*) that Tunn is Elliptical above and Circular below; and to be Gauged, as there directed: And the Dripp or Fall of either, to be found, according to the Rule of the foregoing *Problems*.

2. *Direct.*

2. *Direct.* Of Conical Tunns, some (having great inequality of their extream Diameters) are far remote from a Cylinder; (as in the First, Second, Third, Fourth, and Fifth *Examples*;) others (having lesser difference of extream Diameters) approach nigher to a Cylindrical Figure, as in the Sixth, Seventh, and Eighth *Examples.*

3. *Direct.* Note here, when a round Tunn is exactly Cylindrical, that is, when all its Diameters are equal; that as the Cyclometrical Table Inches the same by meer ocular inspection; so when a Conical Tunn hath small difference of its extream Diameters, (as in the Sixth, Seventh, and Eighth *Examples*) the same is Inched and Gauged with far more facility, than others, which further recede from a Cylindrical Figure, as in the First, Second, Third, Fourth, and Fifth *Examples.*

4. *Direct.* In the Gauging of such Conical Tunns, as have great inequality of their extream Diameters, as in the First *Example, &c.* following. Having, according to the Rule of the last *Problem*, found the Tunns Fall, and (according to precedent Directions) taken the Vessels Diameters (in Inches and Decimal Parts) in the midst of each Foot, half Foot, or Inch of the Depth, (according as you would be more or less precise and exact;) insert them orderly and distinctly in your Book or Paper; (beginning always above, and descending gradually to the Tunns bottom:) Then by the Rule in 39

Probl.

Probl. find the common Subducend, or Addend, for conſtituting the propoſed Tunns decreaſing or increaſing Diameters, from Foot to Foot, half Foot to half Foot, or Inch to Inch of the Depth; whereby the Tunns ſeveral Diameters will (by very ſhort and facile Calculation) be had.

5. If the propoſed Tunns ſeveral Diameters (ſo Inſtrumentally taken as aforeſaid) do agree with their Arithmetical Calculations, or be very nigh thereunto; it is a confirmation that the Veſſel is Conical; and that the Anſwer or Concluſion, grounded upon, or inferr'd from thoſe Diameters, is genuine and practically true; As in the firſt *Example*; where the Diameters are taken in the midſt of each Foot of the Tunns Depth; the Anſwer is too little only by .543 of an Ale Gallon, being $\frac{1}{2}$ a Gallon and the $\frac{1}{3}$ of a Pint in the Tunns Capacity = above 74 Beer Barrels. But in the ſecond *Example*, where the Diameters are taken in the midſt of each half Foot, the Anſwer wants only .1818 of a Gallon, (being not 1 $\frac{1}{2}$ Pint) of the Tunns Capacity: Which ſaid ſmall Errour, will always be in favour of the Brewer: And by how much any Conical Tunn differs more or leſs from a Cylinder, and hath a greater or leſſer Depth, ſo this Errour will be more or leſs; though on the right hand, and very ſeldom (if ever) ſo much as 3 Quarts in a Tunn of 150 Barrels, (in Practice not conſiderable:) Which ſmall Errour nevertheleſs (by a little more

more labour) is alſo removed; as in the Third *Example*, where the Diameters are taken in the midſt of every Inch of the Tunns Depth; the Anſwer ſhall not want half a ſmall ſpoonfull of the truth, or ſhall be found infinitely nigh the truth, by whomſoever ſhall candidly examine the ſame.

6. *Direct.* Having (according to foregoing Inſtructions) Inſtrumentally and Arithmetically found a Conical Tunns Diameters in the midſt of every Foot, half Foot, or Inch of its Depth, and orderly inſerted them, (beginning at the uppermoſt Diameter, and deſcending gradually to the loweſt) then from the Cyclometrical Table for Ale-meaſure inſert againſt each Diameter its reſpective Circles Area, in Gallons and Decimal Parts; which reduces the whole Veſſel (in the firſt Caſe) into ſo many Cylinders, as the Tunns Depth contains Feet: (In the ſecond Caſe) into ſo many Cylinders, as half Feet; and (in the third Caſe) into as many Cylinders, as there are Inches in the Tunns Depth: ſhewing (*propè verum*) how many Ale Gallons each Inch, in thoſe reſpective Feet or half Feet, may contain or hold; which done, in all the three Caſes, add together the Circles Area's (ſo inſerted from the Table) noting down their ſeveral Summs or Totals: Then in the firſt Caſe, the Total of the Area's multiplied by 12 (the number of Inches in one Foot:) In the ſecond Caſe, the Total multiplied by 6, do ſeverally produce (*propè verum*) the

the Tunns whole Capacity in thoſe two reſpective Caſes, either of which (eſpecially the later) is nigh enough the truth for common Uſe and Practice, and how nigh, hath been already inſtanced: But in the third Caſe, the Summ or Total is equal to the Tunns whole Capacity in Ale Gallons, very nigh the truth it ſelf; as in the third and Eighth Examples is made evident and conſpicuous.

The FIRST EXAMPLE ſhall be

Of a Conical Tunn, more than ordinarily remote from a Cylinder, known by the great inequality of its extream Diameters, their difference being equal to $\frac{1}{4}$ of the greater, and $\frac{1}{3}$ of the leſſer Diameter, *viz.*

There is a Conical Tunn, whereof the upper Diameter is = 144 *Inches, the lower Diameter* = 108, *and the Depth* 60 *Inches: To Gauge and Inch this Tunn by the Cyclometrical Table.*

And firſt to Gauge the Tunn upon each Foot.

dep. diff. Diam.

60 : 36 :: 1 : .6 the common Subduc.

6 Inches in $\frac{1}{2}$ a Foot.

3.6 which ſubduct from

gr. Diam. (above) = 144

Tunns Diameter = 140.4 at $\frac{1}{2}$ Foot diſtance from the top.

The

The Feet in the Depth reckoning from the Tunns bottom upwards.	*The Tunns several Diameters from Foot to Foot in Inches.*	*The Circles Area's in Ale Gall. per Inch in each Foot.*	*The Circles Area's in Ale Gall. per Foot.*
5 at $\frac{1}{2}$ a Foot deep	140.4 *per* T.	54.90	658.80
4 at 1 $\frac{1}{2}$	133.2 *per* T.	49.42	593.04
3 at 2 $\frac{1}{2}$	126— *per* T.	44.22	530.64
2 at 3 $\frac{1}{2}$	118.8 *per* T.	39.31	471.72
1 at 4 $\frac{1}{2}$	111.6 *per* T.	34.69	416.28

The Tun holds *prox. per* foot 222.54 2670.48
The Inches in a Foot = 12

Tuns Capacity in Ale Gall. 2670.48 which is too little by half a Gallon and the $\frac{1}{3}$ of a Pint, in above 74 Beer Barrels.

SECOND EXAMPLE.

He that affects more exactneſs and curioſity, may take the Diameters of the Tunn, in the midſt of every half Foot; which in the ſame Tunn is thus performed, *viz.*

The common Subducend, found = .6
The half of 6 Inches = 3

This Product ſubduct = 1.8

From the Tuns greater Diam. being above 144

Tuns Diam. at 3 Inch. depth from the top 142.2

The

The half Feet in the Depth, reckoning from the bottom upwards.		*The ſeveral Diameters from half Foot to half Foot in Inches.*	*The Circles Area's in Ale Gallons per Inch in each half Foot.*
10 at 3	Inches deep.	142.2 *per* T. =	56.3171
9 at 9		138.6 *per* T. =	53.5017
8 at 15		135-- *per* T. =	50.7584
7 at 21		131.4 *per* T. =	48.0874
6 at 27		127.8 *per* T. =	45.4886
5 at 33		124.2 *per* T. =	42.9620
4 at 39		120.6 *per* T. =	40.5074
3 at 45		117-- *per* T. =	38.1252
2 at 51		113.4 *per* T. =	35.8152
1 at 57		109.8 *per* T. =	33.5772

Tunn holds *per* $\frac{1}{2}$ Foot *proximè* 445.1402
The Inches in half a Foot = 6

Tunns Capacity is Ale Gallons 2670.8412; which wants only .1818 of a Gall. being not altogether 1 $\frac{1}{2}$ Pint in the Tunns Capacity of above 74 Barrels.

THIRD EXAMPLE.

To Gauge the aforeſaid Tunn, and to Calculate a Table of its Capacity, from Inch to Inch; whereby any perſon may (by Addition only) ſpeedily and certainly know the quantity of Gallons, remaining in the Tunn (very nigh

nigh the truth it ſelf) at any number of dry Inches, whatſoever, found by the Gauging Rod.

The common Subducend = .6
The half of an Inch = .5
This Product ſubduct = .3

From the greater Diameter being aloft 144

Tuns Diam. at ½ Inch depth from the top 143.7

In the firſt Column of the following Table, is ſet down the Inches in the depth, reckoning from the bottom upwards.

In the ſecond, the Tunns ſeveral Diameters from Inch to Inch in Inches.

And in the third and laſt Column, the Circles Area's in Ale Gallons *per* Inch.

Inches

Inches in Depth.			*Tunns Diam.*			*Circ. Ar.*
60	at	.5 an Inch deep.	143.7	*per*	T.	57.5115
59	at	1.5	143.1	*per*	T.	57.0323
58	at	2.5	142.5	*per*	T.	56.5550
57	at	3.5	141.9	*per*	T.	56.0798
56	at	4.5	141.3	*per*	T.	55.6065
55	at	5.5	140.7	*per*	T.	55.1353
54	at	6.5	140.1	*per*	T.	54.6661
53	at	7.5	139.5	*per*	T.	54.1988
52	at	8.5	138.9	*per*	T.	53.7336
51	at	9.5	138.3	*per*	T.	53.2704
50	at	10.5	137.7	*per*	T.	52.8092
49	at	11.5	137.1	*per*	T.	52.3500
48	at	12.5	136.5	*per*	T.	51.8928
47	at	13.5	135.9	*per*	T.	51.4376
46	at	14.5	135.3	*per*	T.	50.9843
45	at	15.5	134.7	*per*	T.	50.5332
44	at	16.5	134.1	*per*	T.	50.0840
43	at	17.5	133.5	*per*	T.	49.6368
42	at	18.5	132.9	*per*	T.	49.1916
41	at	19.5	132.3	*per*	T.	48.7485
40	at	20.5	131.7	*per*	T.	48.3073
39	at	21.5	131.1	*per*	T.	47.8682
38	at	22.5	130.5	*per*	T.	47.4310
37	at	23.5	129.9	*per*	T.	46.9959
36	at	24.5	129.3	*per*	T.	46.5628
35	at	25.5	128.7	*per*	T.	46.1316
34	at	26.5	128.1	*per*	T.	45.7025
33	at	27.5	127.5	*per*	T.	45.2754
32	at	28.5	126.9	*per*	T.	44.8503
31	at	29.5	126.3	*per*	T.	44.4271

Sum 1525.0095

Inches in Depth.				*Tunns Diam.*			*Circ. Ar.*
30	at	30.5	Inches deep.	125.7	*per*	T.	44.0060
29	at	31.5		125.1	*per*	T.	43.5869
28	at	32.5		124.5	*per*	T.	43.1698
27	at	33.5		123.9	*per*	T.	42.7547
26	at	34.5		123.3	*per*	T.	42.3416
25	at	35.5		122.7	*per*	T.	41.9306
24	at	36.5		122.1	*per*	T.	41.5215
23	at	37.5		121.5	*per*	T.	41.1144
22	at	38.5		120.9	*per*	T.	40.7094
21	at	39.5		120.3	*per*	T.	40.3063
20	at	40.5		119.7	*per*	T.	39.9052
19	at	41.5		119.1	*per*	T.	39.5062
18	at	42.5		118.5	*per*	T.	39.1092
17	at	43.5		117.9	*per*	T.	38.7141
16	at	44.5		117.3	*per*	T.	38.3211
15	at	45.5		116.7	*per*	T.	37.9301
14	at	46.5		116.1	*per*	T.	37.5410
13	at	47.5		115.5	*per*	T.	37.1540
12	at	48.5		114.9	*per*	T.	36.7690
11	at	49.5		114.3	*per*	T.	36.3860
10	at	50.5		113.7	*per*	T.	36.0050
9	at	51.5		113.1	*per*	T.	35.6260
8	at	52.5		112.5	*per*	T.	35.2490
7	at	53.5		111.9	*per*	T.	34.8740
6	at	54.5		111.3	*per*	T.	34.5010
5	at	55.5		110.7	*per*	T.	34.1301
4	at	56.5		110.1	*per*	T.	33.7611
3	at	57.5		109.5	*per*	T.	33.3941
2	at	58.5		108.9	*per*	T.	33.0292
1	at	59.5		108.3	*per*	T.	32.6662

Ths Sum 1146.0128

In the Tunn, when 30 Inch. are wet 1146.0128
And when 30 Inches only are dry, there wants ——— 1525.0095

So the Tunns whole Capacity = 2671.0223 which is 74 Beer Barrels, 6 Gallons, and 1 $\frac{1}{2}$ Pint, *proximè*; allowance being made for the Tunns Fall.

Note also, That when the dry Inches are only 12, there remains in the Tunn Ale Gall. 2012.0727; when 6 Inches dry, there are in the Tunn Ale Gallons 2333.1018; and when $\frac{1}{4}$ of the Depth, *viz.* 15 Inches, are dry, there are in the Tunn Ale Gallons 1857.7590, being a little above $\frac{2}{3}$ of the whole Capacity.

Ale Gall.

658.9486 upon the upper Foot.
337.9205 upon the upper half Foot.
321.0281 upon the ninth half Foot.
593.1112 upon the fourth Foot.
304.5687 upon the eighth half Foot.
288.5425 upon the seventh half Foot.
530.7393 upon the third Foot.
272.9497 upon the sixth half Foot.
257.7896 upon the fifth half Foot.
471.8325 upon the second Foot.
243.0630 upon the fourth half Foot.
228.7695 upon the third half Foot.
416.3907 upon the first Foot.
214.9090 upon the second half Foot.
201.4817 upon the first half Foot.

Admit a Brewer have a Guile of ſtrong Beer in the Tunn (laſt mentioned and Tabulated): And the Meſſenger or Gauger (obſerving that all the Worts are let down) doth by his Gauging Rod find five dry Inches: Now to find (by Addition only) how many Gallons he is to return for this Brewers length.

It is given by the Table, that when the dry Inches are only 6, there then remains in the Tunn —	Beer Gall. 2333.1018
And that the Tunn holds upon the next, or 55*th* Inch from the bottom, — — — — —	55.1353
So there remains in the Tunn	2388.2371

Which (if he cannot multiply or divide) he may by a Table of Reduction of Gallons, *&c* into Barrels, (to be had about him) find to be Returned into the Office of Exciſe —	Bar.	Fir.	Gal.	Pin.
	66	1	3	$1\frac{1}{4}$

Which ſaid Return will ſtand the Teſt of the moſt ſcrutinous Examination.

And thus of the reſt, *&c.*

The FOURTH EXAMPLE shall be

Of a Conical Tunn, nigher to a Cylindrical Form; the difference of the Extream Diameters being not so much as the $\frac{1}{7}$ of the greater, or as the $\frac{1}{6}$ of the lesser Diameter, *viz.*

There is a Conical Tunn, whereof the upper Diameter is 190 *Inches, the lower,* 160, *and the Depth* = 60 *Inches: To Gauge and Inch this Tunn by the Table: And first upon each Foot of its depth.*

Depth diff.Diam.

60 : 30 :: 1 : .5 common Subd.

Mult. *per* $\frac{1}{2}$ of a Foot, to wit, 6 Inches.

3 which subduct

From the upper & gr.Diam. 190

Tunns Diam. at 6 Inch distance from the top } = 187

The Feet in the Depth, reckoning from the Tunns bottom upwards.	*The Tunns ſeveral Diameters from Foot to Foot in Inches.*	*The Circles Area's in Ale Gall. per Inch in each Foot.*
5 at $\frac{1}{2}$ a Foot deep.	187 *per* T. =	97.393
4 at $1\frac{1}{2}$	181 *per* T. =	91.243
3 at $2\frac{1}{2}$	175 *per* T. =	85.294
2 at $3\frac{1}{2}$	169 *per* T. =	79.546
1 at $4\frac{1}{2}$	163 *per* T. =	73.998

Tunn holds *prox. per* Foot Ale Gall. 427.474
The Inches in a Foot = 12

The Tunns Capacity in Ale Gall. 5129.688, which is too little by .466 of a Gallon, being not $3\frac{3}{4}$ Pints, in $142\frac{1}{2}$ Beer Barrels.

FIFTH EXAMPLE.

Such as deſire more exactneſs, may take the ſame Tunns Diameters in the midſt of every half Foot.

The common Subducend = .5 of an Inch,
Mult. *per* the half of 6 Inches, = 3

1.5 ſubduct

from the gr. Diam. being aloft 190

Tuns Diam. at 3 Inch. from top 188.5

The half Feet in the Depth, reckoning from the bottom upwards.	*The ſeveral Diameters from half Foot to half Foot in Inches.*	*The Circles Area's in Ale Gallons per Inch in each half Foot.*
10 at 3 Inches deep.	188.5 *per* T. =	98.9609
9 at 9	185.5 *per* T. =	95.8360
8 at 15	182.5 *per* T. =	92.7612
7 at 21	179.5 *per* T. =	89.7366
6 at 27	176.5 *per* T. =	86.7621
5 at 33	173.5 *per* T. =	83.8378
4 at 39	170.5 *per* T. =	80.9636
3 at 45	167.5 *per* T. =	78.1395
2 at 51	164.5 *per* T. =	75.3655
1 at 57	161.5 *per* T. =	72.6417

Tunn holds *per* $\frac{1}{2}$ Foot *proximè* 855.0049
The Inches in half a Foot = 6

Tunns Capacity is Ale Gallons 5130.0294; which wants of the true Capacity .1246 of a Gallon, being not 1 Pint in 142 $\frac{1}{2}$ Beer Barrels; which Errour, I preſume, is very inconſiderable in Practice.

A SIXTH EXAMPLE ſhall be

Of a Conical Tunn, ſo nigh a Cylindrical Form, that the leſſer Diameter is above 30 times the difference of the extream Diameters, *viz.*

There is a Conical Tunn, whereof the upper Diameter is 215 *Inches, the lower* = 209.6, *and the Depth* 54 *Inches.*

For the Gauging and Inching all Tunns, that are nigh a Cylinder, the moſt compendious Rule (by the Tables aid) is this, viz.

Foraſmuch as an Arithmetical Mean Proportional, between any two known Diameters of any Conical Tunn whatſoever, is equal to that Tunns true Diameter in the midſt of the depth, between thoſe Diameters: In this and all ſuch Caſes therefore, take ſuch a Mean between the Extream Diameters given, which being found, is always equal to the Tunns Diameter in the midſt of its depth, and reduces the Veſſel *(quam proximè)* into a Cylinder: Then this Diameter being found in the Cyclometrical Table for Ale Meaſure, ſhews its Circles Area in Ale Gallons, which *(propè verum)* is equal unto what the Veſſel may hold upon each Inch; Laſtly, this Area (ſo found) being multiplied by the Depth, produces the whole Veſſels Capacity, very nigh the truth; but what ſmall errour there is, will always be in defect.

The leſſer Diameter = 209.6
The Semidiff. of extream Diameters = 2.7
The Mean Diam. = that of the Tun in the midſt of its depth, which is = 212.3

Ale

Ale Gall. *per* Inch.
212.3 *Per* Table = 125.529
Tunns Depth = 54

502116
627645

The Tunns Capacity in Ale Gall. = 6778.556

Which wants only .317 of a Gallon (being 2 Pints and $\frac{1}{2}$) of the true Capacity, that is 188 Beer Barrels and nigh $\frac{1}{3}$.

Which Errour I leave, as inconsiderable in Practice, in matters of this kind.

SEVENTH EXAMPLE.

Admit there be a Conical Vessel, whose upper Diameter is 216 *Inches, the lower* = 112, *and the Depth only* 10 *Inches:*

To Gauge and Inch this Vessel by the Table.

Inches.
The lesser Diameter = 212
Semidifference of the Extream Diameters = 2

Meam Diameter = Tunns Diameter in midst of its depth, which is } = 214

Ale

Ale Gall. *per* Inch.
214 *Per* Table = 127.547
Tunns Depth = 10

The Tunns Capacity in Ale Gall. 1275.470 allowance being made for the Fall.

Which wants .032 of a Gallon (being only the $\frac{1}{4}$ of a Pint) of the Veſſels true Capacity, 35 Beer Barrels, 1 Firkin, 6 $\frac{1}{2}$ Gall. *ferè*; an Errour (in theſe Caſes) reckoned trivial.

EIGHTH EXAMPLE.

To Gauge this Veſſel more exactly from Inch to Inch, and to Tabulate the ſame, ſo that any perſon may (by Addition only) ſpeedily and certainly find out the quantity either of Gallons, or of Beer Barrels, Firkins, Gall. *&c.* remaining in the Veſſel (very nigh the truth it ſelf) at any number of dry Inches he ſhall find by the Gauging Rod.

dep. diff. diam.
10 : 4 :: 1 unto the common ſubducend .4
The half of an Inch = .5

Subduct .2

from the greater Diam. being aloft 216

The Veſſels Diameter at half an Inch depth from the top — 215.8 Inch.

The

The Inches in the Depth, reckoning from the bottom upwards.			*The Tunns ſeveral Diameters from Inch to Inch of the Depth in Inches.*		*Circles Area's in Ale Gall. per Inch.*
10	at	.5	215.8 *per* T. =		129.701
9	at	1.5	215.4 *per* T. =		129.221
8	at	2.5	215— *per* T. =		128.742
7	at	3.5	214.6 *per* T. =		128.263
6	at	4.5	214.2 *per* T. =		127.785
5	at	5.5	213.8 *per* T. =		127.308
4	at	6.5	213.4 *per* T. =		126.832
3	at	7.5	213— *per* T. =		126.357
2	at	8.5	212.6 *per* T. =		125.883
1	at	9.5	212.2 *per* T. =		125.410

Inches deep, of an Inch deep.

Veſſels true Capacity is Ale Gall. 1275.502 ; being 35 Barrels, 1 Firkin, 6 Gall. $\frac{1}{2}$ *ferè.*

Remaining in the Veſſel at each Inches Altitude of Liquor, viz.

Ale Gall. *&c.*

At 1 dry Inch — 1145.801
At 2 dry Inches — 1016.580
At 3 dry Inches — 887.838
At 4 dry Inches — 759.575
At 5 dry Inches — 631.790
At 6 dry Inches — 504.482
At 7 dry Inches — 377.650
At 8 dry Inches — [illegible]
At 9 dry Inches — [illegible]5.410

Admit

Admit a Brewer hath this Week cleanſed three Guilds of Beer from this Veſſel; there being at the firſt Guild found by the Gauging Rod 3 dry Inches, at the ſecond · ½, and at the third Guild 3 ½ dry Inches: What return may be made into the Office; ſo as to be without loſs either to the Brewer, or to the Commiſſioners for that Affair?

The Work.

The firſt Guild, *September* 12. *Anno* 1672.

It appears by the Table, that at 3 dry Inches, there remains in the Veſſel for the firſt Guild }	Ale Gall. 887.838—
The half of the third Inch downwards is equal ——— }	64.371
The ſecond Guild *Septemb.* 14. =	952.209—
It appears alſo that at 4 dry Inches there are ——— }	759.575
The half of the ſeventh Inch is equal to ——— }	64.132
The third Guild on *Sept.* 16. =	823.707—
So the Weeks Return =	2663.754

Which the *Supraviſor* will find he may without one Farthings wrong to the Brewer return 74 Beer Barrels.

And

And thus of any other Tunns, Tables may be Calculated, either in Gallons, or in Barrels and Gallons, to the Capacity of the unskilful, and ſo as to be without a Quart Errour in a Tunn of 200 Barrels.

The NINTH EXAMPLE ſhall be

Of a Tunn, which (though to the Eye, ſeems round) is by help of the ſliding Ruler, found Elliptical above and below.

As for INSTANCE.

There is a Tunn, Elliptical, both at top and bottom; its Axe or greater Diameter = 216 *Inches, the leſſer Diameter, as well above as below,* = 212, *and the Depth* 54 *Inches: How many Ale Gallons may this Veſſel hold upon each Inch of the Depth; and how many, the whole Tunns Capacity?*

The Rule.

An Arithmetical Mean Proportional between the Axe and leſſer Diameter given, reduces the Veſſel *(propè verum)* into a Cylinder; which Arithmetical Mean Diameter therefore (being found) doth by inſpection of the Table for Ale Meaſures ſhew the Circles Area, or number of Ale Gallons, the Tunn may hold upon an Inch, (nigh enough for practical Uſe); from whence

Admit a Brewer hath this Week cleanſed three Guilds of Beer from this Veſſel; there being at the firſt Guild found by the Gauging Rod 3 dry Inches, at the ſecond 3 $\frac{1}{2}$, and at the third Guild 3 $\frac{1}{2}$ dry Inches: What return may be made into the Office; ſo as to be without loſs either to the Brewer, or to the Commiſſioners for that Affair?

The Work.

The firſt Guild, *September* 12. *Anno* 1672.

It appears by the Table, that at 3 dry Inches, there remains in the Veſſel for the firſt Guild } Ale Gall. 887.838—

The half of the third Inch downwards is equal } 64.371

The ſecond Guild *Septemb.* 14. = 952.209—

It appears alſo that at 4 dry Inches there are } 759.575

The half of the ſeventh Inch is equal to } 64.132

The third Guild on *Sept.* 16. = 823.707—

So the Weeks Return = 2663.754

Which the *Supraviſor* will find he may without one Farthings wrong to the Brewer return 74 Beer Barrels.

And thus of any other Tunns, Tables may be Calculated, either in Gallons, or in Barrels and Gallons, to the Capacity of the unskilful, and ſo as to be without a Quart Errour in a Tunn of 200 Barrels.

The NINTH EXAMPLE ſhall be

Of a Tunn, which (though to the Eye, ſeems round) is by help of the ſliding Ruler, found Elliptical above and below.

As for INSTANCE.

There is a Tunn, Elliptical, both at top and bottom; its Axe or greater Diameter = 216 *Inches, the leſſer Diameter, as well above as below,* = 212, *and the Depth* 54 *Inches: How many Ale Gallons may this Veſſel hold upon each Inch of the Depth; and how many, the whole Tunns Capacity?*

The Rule.

An Arithmetical Mean Proportional between the Axe and leſſer Diameter given, reduces the Veſſel *(propè verum)* into a Cylinder; which Arithmetical Mean Diameter therefore (being found) doth by inſpection of the Table for Ale Meaſures ſhew the Circles Area, or number of Ale Gallons, the Tunn may hold upon an Inch, (nigh enough for practical Uſe); from whence

whence the Veſſels whole Capacity cannot be unknown.

Inches.

The leſſer Diameter = 212

The Semidifference of the Diameters = 2

The Arithmetical Mean Diameter = 214

Area in Ale Gall. *per* wet Inch.

214 *per* Table = 127.5464

The Depth 54

5101856

6377320

Reſpon. Tunns Capacity Ale Gall. 6887.5056; which is nigh half a Gallon too much, in the Tunns Capacity, (being above 191 Beer Barrels;) this Errour alſo is not conſiderable in Practice, but in this Caſe will always be in exceſs.

He that inſiſts upon more exactneſs, may anſwer the Queſtion by the Rule of the Eleventh *Problem.*

The TENTH EXAMPLE ſhall be

Of a Tunn, which by the ſliding Ruler is found Elliptical above, and Circular below.

As

As for INSTANCE.

There is a Tunn, Circular below, and Elliptical above; the Diameter of the Circular Baſe = 217 *Inches, the Axe, or greater Diameter of the Elliptical Baſe,* = 216 *Inches, the leſſer Diameter* = 214, *and the Depth* = 50 *Inches: How many Ale Gallons may this Veſſel hold upon an Inch, and how many are the Tunns whole Capacity?*

The Rule.

Take two Arithmetical Mean Proportionals, the one between the Ellipſes Axe, and leſſer Diameter given; and the other, between that Mean, (ſo found) and the Diameter of the Circular Baſe: Which done, the Mean Diameter (laſt found) reduces the Veſſel *(propè verum)* into a Cylinder; and being ſought in the Cyclometrical Table of Ale Meaſure; you have that Circles Area in Ale Gallons; which is equal unto what the Tunn holds upon an Inch, *proximè verum*; which therefore being multiplied by the Tunn or Veſſels depth, produces its Capacity in Ale Gallons, very nigh the truth; where note alſo, that what ſmall Errour there is, will always be in exceſs.

The

Inches.

The Ellipses lesser Diameter = 214
Its Semidifference from the Axe = 1

Its Mean Diameter = 215
This Means Semidifference from the Diameter of the Circular Base = 1

The Diameter of a Mean Circle = 216

Per Inch.

216 *Per* Table = Ale Gallons 129.9416
Tunns Depth = 50

Resp. Tunns Capacity Ale Gallons 6497.0800 which is only 3 Quarts too much in 180½ Beer Barrels; an Errour not much considerable in Practice of this kind.

Such as desire more exactness, may work the Question by the Rule of the 13 *Problem.*

The like Rule serves if the Vessel be Circular above, and Elliptical below.

XLII. Probl.

If in a Conical Tunn, whose lower Diameter is 108 *Inches, there do remain* 2671.022 *Ale Gallons, and the Diameter of the Liquors superficies be* 144 *Inches: what is the Altitude or Depth of the Liquor, or how many are the Tunns wet Inches?* Fig. 9.

The Rule.

The Square of the Summ of the Diameters, less their Rectangle, Multiply (the remaining Liquor being given in Beer or Ale Gallons) always by .000928366-|-; if in Beer Barrels, by .00002578797-|-; if in Ale Barrels, by .00002901146; and if in Wine Gallons, by .00113331-|-: Which done, the Product so accruing, divided by the remaining Liquor proposed, quotes the Liquors Altitude, or wet Inches sought,

But for avoiding Division, a Multiplicator for the abovementioned Product may be had by the Rule in *Probl.* 61. Yet this not falling within the verge of *Practical Gauging*, but rather a matter of Curiosity for tryal of Skill, should not be pretended unto, unless by such as are expert and ready at Division, *&c.*

Multiplicat. for Ale Gall. = .000928366+
q. of Summ Diam. + their Rectang. 47952

Their Product = 44.51704

For which the Multiplicator = .02246331
Remaining Liquor, Ale Gall. 2671.022

Respons. Inches 59.99999, *&c.*

Which according to the Doctrine of Decimals is = 60 Inches for the Altitude, Depth, or wet Inches of the Tunn, sought.

XLIII. Probl.

Fig. 9. *There is a Conical Tunn, as* a, b, d, c, *the upper Diameter* a, b, = 144 *Inches, the lesser Diameter* c, d, = 108, *and the Depth* n, o, = 60 *Inches: What is this Tunns side or slant heighth?*

The Rule.

The Square Root of the Summ of the Squares of the perpendicular heighth or depth, and of the Semidifference of Diameters, is equal to the slant heighth, sought.

144
108
36
18
18
144
18
324
60
60
3600
324
3924 : 3,5937290
1,7968645 *Resp.* 62.64185 + the
Inches.
ſlant heighth, ſought.

XLIV. Probl.

Having the perpendicular Altitude and ſide line of the Fruſtum of a Cone, together with its Diagonal; to find the Fruſtums Diameters and liquid Capacity.

There is a Conical Glaſs, as, r, s, z, *having two Hoops* u, a, o, *and* m, c, n; *the quantities of the Diameters at theſe Hoops cannot be taken; but the diſtance of the Hoops in perpendicular heighth from* c *to* a, *is found* 2.9 *Inches, the ſlant heighth or ſide line of* u *above* m, *or* o *above* n, *is* 3.198828, *and the Diagonal line from* u *to* n, *or from* o *to* m, *is* 3.732627: *Now this Glaſs ſtanding Horizontally upon the Table, with ſo much Wine in it, as reaches to the lower Hoop* m, c, n, *the queſtion is, How much liquor muſt be added, ſo as to raiſe the ſuperficies of the Liquor, to the upper Hoop* u, a, o?

The Rule.

1. From the quadrupled Square of the Diagonal ſubduct the q. of the doubled heighth of the Fruſtum given; which done, the Square Root of that Remainder, is equal to the Summ of the Diameters, *u, a, o,* and *m, c, n.*

2. The Difference of the Squares, of the perpendicular heighth and ſlant heighth, is equal to the Square of the Semidifference of the Diameters; whereof the Square Root is their Semidifference.

3. The Semiſumm of the Diameters, more their Semidifference, is equal to the greater of them, *viz.* at *u, a*; and the Semiſumm, leſs the Semidifference, ſhews the leſſer Diameter, *viz.* at the Hoop *m, c, n.*

4. Mul-

4. Multiply the Fruſtums perpendicular Altitude, given, (if the Capacity be ſought in Wine Pints) by .0143003-+-, and that Product, by the Square of the Summ of the Diameters, leſs their Rectangle, and that Product anſwers the Queſtion in Wine Pints.

The Square of the Diag. 3.732627 = 13.9325 *Fig.* 17.
4

Its quadrupled Square = 55.73

Square of 5,8 the doubled heighth = 33.64

The Remainder = 22.09

Inches.

Its Square Root 4.7 = Summ of the Diameters *u*, *a*, *o*, and *m*, *c*, *n*.

The Square of *u*, *m*, 3.198828 = 10.2325

Square of *a*, *c*, perpendicular 2.9 = 8.41

1.8225

Inches.

Whereof Square Root 1.35 = Semidiff. Diam.
½ ſum Diam. afore found 2.35

So the Diam. are found { 3.7 = Diam. *u*, *a*, *o*,
1 = Diam. *m*, *c*, *n*.

.0143003 + } the Multiplicat. for Wine Pints.
2.9 the Fruſtums Alt. *a, c.*

.04147087

18.39 } q. of the Summ of the Diam.—their Rectang.

Reſp. .7626493 of a Wine Pint, being $\frac{31}{40}$ of a Pint, or $\frac{3}{4}$ of a Pint, and $\frac{1}{40}$ of a Pint over.

Some Gallants at a Tavern, drinking Claret in a large Glaſs, one of them ſaid he had then drank twelve times, and ſo much at each draught, as that Glaſs (repreſented by Fig. 17.) *might contain between the Hoops* u, o, *and* m, n; *the Glaſſes Diameter at* u, o, *being* 3.7 *Inches, and* m, n, 1 *Inch; and the Diſtance between thoſe Hoops in perpendicular heighth,* viz. a,c,2.9 *Inch. He laid a Wager of* 5 *l with a Mathematician (in the Company,) that he doth not in one days ſpace, authentickly reſolve him in writing, how many Wine Pints, and Parts of ſuch Pint, he had then ſo drank; computing the ſame according to the rate of* 231 *Cubical Inches to the Gallon.*

The Rule.

This Glaſs being in the form of a Cone, and the ſolidity of a Fruſtum or ſecond Segment of it being required in Wine Pints, ſquare the Summ of the Diameters, and multiply the Diameters by one another, and ſubduct the

later Product out of the former, then multiply the common Multiplicator for Wine Pints in a Cones Fruſtum, *viz.* .01430028, by the Fruſtums Altitude, and that Product again by the Square of the Summ of the Diameters, leſs their Rectangle; which done, the Product (ſo found) is the Queſtions Anſwer in Wine Pints.

The Multiplicator = .01430028
Fruſtums Altitude $a, c, = 2.9$

12870252
2860056

.041470812
q. of Sum Diam.—their Rect. 18.39

373237308
124412436
33176649 6
41470812

.7626482(3268
12

Reſp. 9.1517784 Wine Pints, being $9\frac{3}{20}$ Pints of Wine.

XLV. Probl.

Having a Conical Tunns Dimensions; to find the number of wet Inches, or Altitude of the Liquor, when any assigned part of the Tunns given Capacity, remains therein.

Fig. 9. *Of the Conical Tunn* a, b, c, d, *admitting the greater Diameter* a, b, = 66 *Inches; the lesser Diameter* c, d, = 40; *and* n, o, *the Depth* = 55 *Inches, and the Tunns whole liquid Capacity* = 438.4566 *Ale Gallons: what shall be the Liquors Altitude, or how many shall the wet Inches be, when the Tunn is half full, that is, when* 219.2283 *Gallons, the half of its Capacity, remain in it?*

$a, b, = 66$

$c, d, = 40$

$26 : 40 :: 55 : 84.61538 = z, o.$

$55 = n, o.$

$139.61538 = z, n,$

being the whole Cones Axe.

Pu

Per Cyclometrical Tab. the Area of the Circle at the Cones Base *a, b*, 66 —	Ale Gall. = 12.13189501
$\frac{1}{3}$ of *x, n*, the Cones Axe	= 46.53846
Produces whole Cones Capac.	564.5997
The Tunns Semi-capacity	219.2283
Cones Capacity, less the Tunns Semi-capacity,	= 345.3714

Then by this Analogism:

Co. ar. 7,2482594 :	As 564.5997, the whole Cones Capacity;
5,0034360 :	to the Cube of 46.53846, the $\frac{1}{3}$ of *x, n*:
2,5382864 :	so is 345.3714, the Cones Capacity, less the Tunns Semi-capacity.
4,7899818 :	to the Cube of $\frac{1}{3}$ Alt. *x, i*.
1,5966606 :	39.505783
	118.51734 Altitude *x, i*.
	84.61538 = *x, o*, part Con. Axe cut off.
Respons.	33.90195 Inch. Tuns dep. (*i, 4,*) are wet.

And consequently, 21.09805 for *n, i*, which make the whole Depth 55 Inches, as was proposed.

In-

Inversion of the QUESTION.

Of the Conical Tunn aforesaid, if n, i, *the dry Inches be* 21.09805, *what is the lesser Diameter,* e, i, m, *and how many Ale Gallons are wanting to fill up this Tunn?*

Depth. diff. Diam.

55 : 26 :: 1 : .4727273+
21.09805

Product = 9.97362

66

The Diameter *e, i, m,* = 56.026638

Square of Summ Diameters, less their Rectangle,
= 11192.69634
21.09805

236144.067

Multiplicator for Ale Gallons .00092836 7+

Respons. Ale Gallons 219.2283 wanting to fill up the Tunn, which is $\frac{1}{4}$ the Capacity thereof.

A very nigh Approximation to the same thing, may be made by the Cyclometrical Table.

As for INSTANCE.

Of the Conical Tunn (Inched and Tabulated in Pag. 97 *and* 98,) *the whole Capacity being given* = *Ale Gallons* 2671.0223, *and the Depth* n, o, = 60 *Inches: The Queſtion is, (when this Tunn is juſt half full, or hath in it* 1335.5111 *Gall.) how many the wet Inches are?* Fig. 9.

	Ale Gallons.	wet Inches.
It doth appear by *Pag.* 98, that there remains in this Tunn	1146.0128 at	30
And by *Page* 97, that the Tunn holds	44.4271 upon the	31
	44.8503 upon the	32
	45.2754 upon the	33
	45.7025 upon the	34
So it follows that there are in the Tunn	1326.2681 at	34
Which is leſs than the Semi-capacity by	9.2430 at	.2
Which is nigh the $\frac{1}{5}$ of what the Tunn holds upon the next or 35*th* Inch, 48.1516(5 / 9.2430		
	1335.5111 at	34.2

Reſponſ wet Inches = 34.2. Which

Which upon ſtrict examination will be found very nigh the truth it ſelf.

And thus of any other.

XLVI. Probl.

Fig. 10. *Let* b, z, a, *repreſent an oblique Cone, and* b, a, c, y, *a leaning Tunn, and* y, c, *the ſuperficies of the Liquor on both ſides: let the whole Cones ſolidity be* 4620.146 *Ale Gallons, (according to* 282 *Cubical Inches to the Gallon)* b, y, *one ſide line of the Tunn* 62.641 *Inches, and* a, c, *the other,* 67; *the whole Cones Altitude* z, n, 240 *Inches, and* b, a, *the Diameter of its Baſe* 144 *Inches: What is the ſolidity of the Elliptical Cone* z, c, y, *and how many Ale Gallons is this leaning Tunns Capacity?*

The Rule.

1. To find the whole Cones Hypothenuſal or ſide-line *z, b*; of the Summ of the Squares of the whole Cones Altitude, and of the Semidiameter of its Baſe, extract the Square Root, for the Hypothenuſe, or ſide-line *z, b*; from whence appear *z, y*, and *z, c*.

2. To find a Geometrical Mean Proportional between *z, y*, and *z, c*; multiply them into each other, and of that Product extract the q. Root, which is the Mean Proportional ſought; the which Cube, by multiplying it into it ſelf, and

and that Product again by the ſame Mean Proportional, and reſerve the ſame.

3. As the Cube of the whole Cones Hypothenuſe z, b, is to the Cones liquid Capacity, ſo is the Cube of the Geometrical Mean Proportional between z, y, and z, c, to the ſolidity of the Elliptical Cone z, c, y, which taken out of the whole Cones given Capacity, leaves the leaning Tunns Capacity ſought.

q. of z, n, 240. is 57600
q. of $\frac{1}{2} b, a$, 72. is 5184

Their Summ 62784 { whereof the q. √ 250.5673.

250.5673 the Cones ſide-line z, b.

62.641 the Tunns ſide-line b, y.

187.9263 the ſide-line z, y.

67 the ſide-line a, c.

183.5673 the ſide-line z, c.

z, y, 187.9263 : 2,2739875
z, c, 183.5673 : 2,2637953

Logarithm of Product 4,5377828

Logarithm of Mean Proportional 2,2688914

Log. of Cube of Mean Proport. 6,8066742

Co. Ar.

So.ar. 2,8032268 : As the Cube of *z*, *b*, 250.5673 ;
3,6646558 : to 4620.146 Ale Gall. the Cones Capacity :
6,8066742 : So the Cube of the Geom. Mean betw. *z*, *y*, & *z*, *c* ;
3,2745568 : to 1881.727 Ale G. Cap. Ell. Co. *z*, *c*, *y*.

And therefore 2738.419 the leaning Tunns Capacity.

Proof of the Work by reducing this Oblique Cone, into a Right Cone.

The Rule.

1. Find a Geometrical Mean Proportional between *z*, *y*, and *z*, *c*, which is equal to *z*, *y*, or *z*, *c*, in a Right Cone.

2. Find the Quantity of the Diameter *y*, *c*, when in a Right Cone, by this Analogism, *viz.* As the Cones whole Hypothenuse *z*, *b* ; is to its Base, *b*, *a* : So is the Geometrical Mean Proportional, (last found,) *viz.* *z*, *y* ; to the Diameter *y*, *c*, in a Right Cone.

3. Then to find the Altitude or Depth of the Tunn : The Square Root of the Difference of the Squares of the Right Cones Hypothenuse, from the top of the Tunn upwards, *viz.* *z*, *y*, (last found) and, of the Right Cones Semidiameter *y*, *c*, is equal to the Tunns Depth or Altitude.

Now

Now the leaning Tunn being thus reduced to an upright Tunn, and the leſſer Diameter, y, c, found 106.743 *Inch. and the Depth,* 62.0993; *the greater Diameter remaining,* 144: *How many Ale Gallons is this Tunns Capacity?*

co.ar. 6,9677195 : As 1077.161 ;
1,7930871 : to 62.0993 Tunns Depth :
4,6767033 : So 57501.05, q. of Summ
——— Diam. — Rectangle ;
3,4375099 : to 2738.420 Ale Gallons,
Tuns Cap.

And conſequently 1881.726 for the Elliptical Cone, as was propoſed.

XLVII. PROBL.

Of Gauging the Wort, remaining in a Cooling Back, partly empty.

As for INSTANCE.
(In the Back mentioned *Probl.* 10.)

Admit a Meſſenger, or Gauger, finds the deepeſt place of the remaining Wort, 5 *Inches; in another place,* 4.3 ; *in another place,* 4.1 ; *in another,* 3.7 ; *and in the ſhalloweſt place, only* 3.4 *wet Inches: At what certain Depth, is the Officer to reckon this Wort?* Fig. 16.

A

A General Rule, observable in such Cases, viz.

Divide the Summ of the wet Inches (found in all the Places, be the same more or fewer) by the number of the Terms, or Places they were taken at; or, multiply the said Summ by the proper Multiplicator for the number of Terms; and that Quotient, or Product, shews the Worts true Depth, to be reckon'd, and made use of: Or, in case the deepest and shalloweſt Places be known, an Arithmetical Mean between them, is the Depth sought.

Where Note, That .1, *&c.* or any small part of an Inch, may in this Case (as is humbly conceived) be allowed the Brewer, in consideration of Wash, Leakage, *&c.*

	Inches.
	5
	4.3
	4.1
	3.7
	3.4
The Summ	20.5
Multiplic. for 5, the number of Terms,	= .02
	4.1 being $4\frac{1}{10}$ Inches.

Respons. The remaining Worts true Depth to be reckon'd, is only 4 Inches.

XLVIII.

XLVIII. PROBL.

To Gauge and Inch a Brewers Copper, with defalcation for the rising Crown. *Fig.* 11.

As for EXAMPLE.

There is a Copper, which (by reason of the Curvity of the Sides, from the Top, to the Diameter, passing by the Vertex of the Crown) is judged Spheroidal; but the lower part (for the streightness of the sides) accounted Conical; and the Crown, Spherical; or a Section of a Sphear.

The Dimensions are these, viz.

	Inches.
a, *b*, the upper Diameter	= 147.5
u, *i*, the Diameter passing by the Vertex of the Crown	= 123.5
n, *x*, the Depth from the top to the Vertex of the Crown	= 34.5
c, *d*, the Diameter at the bottom	= 109
o, *x*, the Crowns Altitude	= 11.5

1. *To find the liquid Capacity of the lower, or Conical part of the Copper, to wit, from the Diameter, passing by the Vertex of the Crown, to the bottom, that is, of* u, x, i, d, o, c, *as if there were no Crown.*

Vide Probl. 41 *Example* 6.

c, d, the Diameter at bottom = 109
Semidifference of extream Diam. = 7.25

Their Summ 116.25

The which, because of a considerable inequality of the extream Diameters 147.5 and 123.5, reckon to be a small matter more, to wit, 116.3.

Then the Area of that Circ. whose Diameter is — } Inches. 116.3 *per* T. is = Ale Gall. 37.6704

The Crowns Altitude = 11.5

1883520
376704
376704

Capacity of the Conical Frustum 433.20960

Vide Probl. 8. *Rule* 2.

2. *Having* c, d, *the Diameter of the Copper,* viz. *of the Base* 109, *and the Altitude* 0, x, = 11. *Inches: To find the liquid Capacity of the Coppers Crown, (as the Section of a Sphear,) without knowledge of the Sphears Axe.*

q. of *c, d*, = 11881
$\frac{4}{3}$ of q. of Alt. *o, x*, = 176.333

Their Summ 12057.333

12057

12057.333

The Altitude *o, x,* = 11.5

60286665
12057333
12057333

138659.3295

Multiplicator for Ale Gall. = .0013926+

8319559770
27731.86590
1247933.9655
4159779885
1386593295

Ale Gallons = 193.0969822617 0 =

= The Capacity of the Coppers Crown; being 5 Beer Barrels, 1 Firkin, and 4 Gallons.

Then the Crowns Capacity, being ſubducted out of the Conical Fruſtums Capacity, *viz.* out of *u, x, i, d, o, c,* above found, 433.2096 Ale Gall. there remains Ale Gall. 240.1126, *&c.* for *u, c, x,* and *i, d, x,* being of Beer Barrels 6 $\frac{1}{2}$, and 6.1126 Gallons of Liquor, juſt to ſave or cover the Coppers Crown, where the Diameter *u, i,* is given 123.5 Inches.

3. *To find the liquid Capacity of the upper, or Spheroidal part of this Copper, to wit, of* a, n, b, i, x, u.

	Ale Gall.
The Area of that Circle, whose Diameter is 147.5 Inches, *per* Table, doubled, ——	= 121.1866
And of 123.5 *per* Table =	42.4790
Their Summ	163.6656
Whereof the $\frac{1}{3}$ is the Area of the Mean Circle, shewing the Capacity of each Inch, *quam prope*, ——	= 54.5552
The Depth from the top of the Copper to the Vertex of the Crown ——	= 34.5
	2727760
	2182208
	1636656
When the Crown is saved, there wants, to fill it up,	1882.1544

Then it being found, that 240.1126 were in the Copper, when the Crown was saved, it appears that the Coppers whole Capacity is 2122.2670, Gall. being of Beer Bar. 58 $\frac{1}{4}$, and 7.2670 Gallons, which is nigh 66 $\frac{1}{4}$ Ale Barrels.

4. *To Inch this Coppers upper Part; to wit, from the top, to the Vertex of the Crown, being Spheroidal. See the Rule following the Work.*

The Coppers several Depths, reckoned from the top to the Vertex of the Crown, 34 ½ Inches.		*The Area's, or Contents upon each Inch in Ale Gallons.*	*Differences of Area's.*
At .5	an Inch from the top.	60.58981	com. subd.
At 1.5		60.55938	.03043
At 2.5		60.49852	.06086
At 3.5		60.40723	.09129
At 4.5		60.28551	.12172
At 5.5		60.13336	.15215
At 6.5		59.95078	.18258
At 7.5		59.73777	.21301
At 8.5		59.49433	.24344
At 9.5		59.22046	.27387
At 10.5		58.91616	.30430
At 11.5		58.58143	.33473
At 12.5		58.21627	.36516
At 13.5		57.82068	.39559
At 14.5		57.39466	.42602
At 15.5		56.93821	.45645
At 16.5		56.45133	.48688
At 17.5		55.93402	.51731

Summ 1061.12991

The Coppers several Depths.	*The Area's in Ale Gallons.*	*Differen. of Area's.*
At 18.5 an Inch from the top.	55.38628	.54774
At 19.5	54.80811	.57817
At 20.5	54.19951	.60860
At 21.5	53.56048	.63903
At 22.5	52.89102	.66946
At 23.5	52.19113	.69989
At 24.5	51.46081	.73032
At 25.5	50.70006	.76075
At 26.5	49.90888	.79118
At 27.5	49.08727	.82161
At 28.5	48.23523	.85204
At 29.5	47.35276	.88247
At 30.5	46.43986	.91290
At 31.5	45.49653	.94333
At 32.5	44.52277	.97376
At 33.5	43.51858	1.00419
At 33.75	21.24197	

Summ 821.00125
1061.12991

The Capacity = 1882.13116, being too little only, by the ¼ of a Pint, .02430.

The

The Coppers several Depths, reckoned from the Vertex of the Crown upwards to the Curb.	*Ale Gallons in the Copper at each wet Inch from the Vertex of the Crown.*	*Being Beer Bar.*	*Firkins.*	*Gallons.*
When the wet Inches are 46	2122.2441	58	3	7
At 45 wet Inches	2061.6543	57	1	0
At 44 wet Inches	2001.0949	55	2	3
At 43 wet Inches	1940.5964	53	3	5
At 42 wet Inches	1880.1892	52	—	8
At 41 wet Inches	1819.9037	50	2	1
At 40 wet Inches	1759.7703	48	3	4
At 39 wet Inches	1699.8195	47	—	7
At 38 wet Inches	1640.0817	45	2	2
At 37 wet Inches	1580.5873	43	3	5
At 36 wet Inches	1521.3668	42	1	—
At 35 wet Inches	1462.4505	40	2	4
At 34 wet Inches	1403.8692	38	3	8
At 33 wet Inches	1345.6529	37	1	4
At 32 wet Inches	1287.8322	35	3	0
At 31 wet Inches	1230.4375	34	—	6
At 30 wet Inches	1173.4993	32	2	3
At 29 wet Inches	1117.0480	31	—	1

The Coppers several Depths.	*Ale Gall. in the Copper.*	*Bar.*	*Firk*	*Gall.*
At 28 wet Inches	1061.1140	29	1	8
At 27 wet Inches	1005.7277	27	3	6
At 26 wet Inches	950.9196	26	1	5
At 25 wet Inches	896.7201	24	3	5
At 24 wet Inches	843.1596	23	1	6
At 23 wet Inches	790.2686	21	3	7
At 22 wet Inches	738.0775	20	2	—
At 21 wet Inches	686.6167	19	—	2
At 20 wet Inches	635.9166	17	2	5
At 19 wet Inches	586.0077	16	1	1
At 18 wet Inches	536.9204	14	1	3
At 17 wet Inches	488.6852	13	2	2
At 16 wet Inches	441.3324	12	1	—
At 15 wet Inches	394.8925	10	3	7
At 14 wet Inches	349.3960	9	2	7
At 13 wet Inches	304.8732	8	1	7
At 12 wet Inches	261.3546	7	1	—
when the crown is just covered, the wet Inches being 11 $\frac{1}{2}$ —	240.1126	6	2	6

To find the Common Subducend, for conſtituting any Spheroidal Veſſels Contents, or ſeveral Capacities, upon each Foot, half Foot, or Inch, of the Depth; and ſo to Inch the ſame throughout, by Addition and Subduction only; (applicable alſo to a Cask, ſtanding on the Head,) &c.

A Brief and Practical Rule may be this, viz.

1. By the Rule upon the 41 *Probl.* take (with a ſliding Ruler) the Veſſels Diameters at top, bottom, and middle; and alſo croſs-wiſe, to ſee if it be Elliptical; then take 2 Diameters in Inches and Centeſſimal Parts, (if need require) *viz.* at 6 Inches, 3, or $\frac{1}{2}$ an Inches diſtance from the greater Diameter; and another, at 18, 9, or 1 $\frac{1}{2}$ an Inches diſtance from the ſaid greater Diameter: Noting down theſe two Diameters in a Book or Paper, the leſſer of them under the greater.

2. Directly againſt theſe two Diameters (if taken in Inches, and Decimal Parts, or Quarters of an Inch) inſert from the Cyclometrical Table their reſpective Circles Area's: Otherwiſe, by Rule in *Probl.* 3. Calculate the proper Area's of the Circles, anſwering to thoſe Diameters; the which being found, and inſerted as above directed; the Difference of thoſe two Circles Area's, *viz.* what the Veſſel holds upon the firſt Foot, half Foot, or Inch, more than upon the ſecond, is the Common Subducend,

pecu-

peculiar to the propoſed Veſſel, for Inching the ſame throughout.

3. The doubled Subducend, taken from the Veſſels Area, or Content, upon the ſecond Foot, half Foot, or Inch; leaveth its Content upon the third Foot, half Foot, *&c.* The Summ of the Subducend, and laſt ſaid Difference, (being the doubled Subducend) deducted out of the Content upon the third Foot, half Foot, *&c.* the Remainder ſhews the Veſſels Content upon the fourth Foot, *&c.* the Summ of the Subducend and laſt mentioned Remainder, ſubducted from the Content upon the fourth, leaveth the Veſſels Area or Content upon the fifth Foot, half Foot, or Inch of the Depth; and after like ſort proceed through the Depth: (which done) In all the three Caſes, add together the Circles Area's or Contents, noting their Summ or Total, the which in the firſt Caſe multiplied by 12, in the ſecond by 6, do ſeverally produce the Veſſels liquid Capac. (near enough for common uſe;) and in the third Caſe, the Total it ſelf is a near approximation to the Veſſels whole liquid Capacity; as appears by the preceding Example.

But in caſe more curioſity and exactneſs be required; then,

(According to the Rules upon the 56 and 58 *Problems*) Calculate the Spheroids Axe, and the propoſed Veſſels two Diameters, in the firſt

first part of the aboveſaid Rule mentioned; as alſo, the proper Area's of the Circles, correſpondent to thoſe Diameters; wherewith finiſh the Work, according to direction of the Rule above-given.

XLIX. PROBL.

Having z, n, *the Axe of a Spheroid, and* D, m, *the Diameter; To find the Spheroids Capacity in Wine or Ale Gallons, either without, or with the Cyclometrical Table.* Fig. 12.

The Rule for finding the ſame, without aid of the Cyclometrical Table.

Multiply the Axe by the Square of the Diameter, and that Product (if the Capacity be ſought in Wine Gallons) multiplied by .002265661—; but if in Ale Gallons, by .001856733—, produces your deſire.

EXAMPLE.

Let z, n, *the Spheroids Axe, be* 67.49402+ *and* D, m, *the leſſer Diameter* = 25 *Inches; To find the Spheroids whole Capacity in Wine Gallons.*

$z, n, = 67.49402+$
Square of $D, m, 25 = 625$

33747010
13498804
40496412

42183.76250
The Multiplicator .00226666 1—

Spheroids Capac. Wine Gall. 95.616 *&c.*

Another way, by the Cyclometrical Table.

$\frac{2}{3}$ of the Spheroids Axe $= 44.996-$
Diam. 25 Inches *per* Tab. Wine Gall. $2.125+$

224989
89992
44996
89992

Spheroids Capac. in Wine Gall. 95.616500
prout ſuprà, quàm proximè.

L. PROBL.

Fig. 12. *Having a Spheroids Capacity, either in Wine or in Ale Gall. together with the quantity of its Axe; To find the leſſer Diameter* D, m.

The

The Rule.

The propoſed Capacity of the Spheroid, being divided by two thirds of its given Axe, or multiplied by its proper Multiplicator, ſhews the Area of the leſſer Diameter, *D, m,* either in Wine or Ale, according to what the Spheroids Capacity was propoſed in; the which being ſought in the Cyclometrical Table, doth in the next Column on the left hand, ſhew the leſſer Diameter, the thing required.

EXAMPLE.

A Spheroids liquid Capacity is 95.616 *Wine Gallons, and its Axe* 67.49402 + *Inches; How much is* D, m, *its leſſer Diameter?*

Proper Mult. for $\frac{2}{3}$ Axe 44.996 = .0222242 +
Spheroids Capacity in Wine Gall. = 95.616

1333452
222242
1333452
1111210
2000178

2.1249891072

Then in the Cyclometrical Table for Wine Gallons, I find 2.1250 Wine Gallons, anſwering to 25 Inches Diameter, which is the Diameter ſought.

LI. PROBL.

Fig. 12. *Having a Spheroids liquid Capacity, viz. 95.616 Wine Gallons, together with its lesser Diameter, D, m, = 25 Inches; To find the quantity of the Spheroids Axe, z, n.*

The Rule.

The proposed Capacity of the Spheroid, being divided by the Area of the lesser Circle, which is given in the Cyclometrical Wine Table, *viz.* against the Diameter 25 Inches, 2.125 Wine Gallons, or multiplied by its proper Multiplicator, *viz.* .4705883 + shews two thirds of the Axe sought, whereunto the half of it self being added, gives the Axe required.

Spheroids Capac. in Wine Gall. = 95.616
Multiplicat. for 2.125 Wine Gall. = .4706 +

573696
6693120
382464

$\frac{2}{3}$ of the Axe = 44.996 *&c.*
Its $\frac{1}{2}$ = 22.498

The Spheroids Axe = 67.4940 Inch.

LII.

LII. Probl.

To cut a Spheroid with a Plane, parallel to the Axe, as d, y, d, *or* u, x, w; *which is to Gauge the Liquor in a Cask (partly empty) lying with the Axe parallel to the Horizon, when the ſuperficies of the remaining Liquor cuts not the Heads of the Cask.* Fig. 13.

The Analogiſms are two, viz.

1. As the Casks length, is unto a Geometrical Mean Proportional between the Summ and Difference of the Boung and Head Diameters; ſo is always (for Wine Gallons) 220.5887— and for Ale Gallons 269.2902+ (being the Quote of 3 divided by 3.1415926, multiplied by 231, or 282,) unto a fourth Proportional, which is univerſal or general for that Cask.

Thus for the Wine Pipe, the fourth Proportional is 81.17449+, for which the proper Multiplicator is = .01231914— for Wine Gallons.

And for the Graves Hogſhead, the fourth Proportional is 81.70674—, for which the proper Multiplicator is = .01223889— for Wine Gallons.

2. As the fourth Proportional (firſt found) viz. 81.17449 for the Pipe, or 81.70674 for the

the Hogshead is unto the Square of the dry Inches, or the Boung Diameters lesser Segment; so is the Remainder of the Boung Diameter, (more that Semidiameter) unto the Casks vacuity, either in Wine or Ale Gallons; when the remaining Liquor reaches exactly to the upper Orbs of the Casks Head; which holds true also in all Cases (of the same Cask) the Liquor not cutting the Heads.

EXAMPLE.

A Canary Pipes Boung Diameter is 29 *Inches, Head Diameter* 23, *and length* 48 *Inches; if the dry Inches be* 3, *how many Wine Gallons wants it of being full, the Liquor reaching exactly to the upper Orbs of the Casks Heads?*

Multiplicator for the Pipe = .01231914—
The Square of 3, the dry Inches = 9

.11087226
Rem. of Boung Diam. + Semidiam. = 40.5

55436130
443489040

Resp. there wants Wine Gall. 4.490326530

LIII. PROBL.

To cut a Spheroid with a Plane, parallel to the leſſer Diameter; having the Spheroids Dimenſions, together with the Quantities of the Diameter of the Fruſtums Baſe, and of the Fruſtums Altitude. Fig. 12.

As for INSTANCE.

Let the Spheroids Axe z, n, *be* 78.80648, D, m, *the leſſer Diameter* = 29 *Inches, and* x, p, æ, *the Diameter of the Fruſtums Baſe,* = 27.62246, *found by Direction* Probl. 59. *(if not given) and the whole Spheroids Capacity (if not given) found by Direction of* Probl. 49. = 150.226 *Wine Gallons, and the Altitude of the Fruſtum,* n, p, = 51.40324 *Inches, whereof the Altitude above the mediety,* viz. C, p, = 12 *Inches; To find the liquid Capacity of the Spheroids Fruſtum* x, p, æ, n, *or of* x, p, æ, z.

The Rule.

1. To the doubled Square of the Spheroids leſſer Diameter *D, C, m,* add the Square of the Diameter *x, p, æ.*

2. That Summ multiply by *C, p,* the Fruſtums Altitude, above the Spheroids Semi-axe, or Mediety.

3. That Product multiplied alwayes by .00113333 1 + for Wine Gallons, or by .0009283666 + for Ale Gallons, produces the Wine or Ale Gallons = the Spheroids Frustum, above the Mediety, *viz.* of *x*, *a*, *m*, *D*, the which being added unto the Hemispheroids Capacity, gives the Capacity of the greater Frustum, or subducted from it, the Remainder is = the lesser Frustums Capacity.

The doubled Square of the lesser Diameter *D*, *m*, 29 Inches ——	= 1682
The Square of the Diameter of the Frustums Base, *x*, *p*, *a*, 27.62246	= 763
The Summ	= 2445
C, *p*,	= 12
	29340
The Multiplicator	.00113333
The Spheroids Frustums Capacity, *x*, *p*, *a*, *m*, *D*, = Wine Gallons	33.252&c
Capac. of Hemispheroid, *D*, *m*, *n*,	75.113
Respons. The Capacity of the Spheroids Frustum *x*, *p*, *a*, *n*,	= 108.365 Wine Gallons.

Another way, by the Cyclometrical Table.

	Wine Gall.
The doubled Area of the Spheroids leſſer Diameter 29 Inches, *D*, *m*,	5.7188
The Area of the Diameter of the Baſe of Fruſtum *x*, *p*, *æ*, 27.62246	2.5942
Tripled Area of the Mean Circle	= 8.3130
$\frac{1}{3}$ of *C*, *p*, Fruſtums Alt. above the Mediety	= 4
Fruſt. Cap. above Mediety, *viz. x p, æ, n*,	33.252
The Hemiſpheroid Capacity *D*, *m*, *n*,	75.113
Reſp. The Capacity of the Spheroids greater Fruſtum *x*, *p*, *æ*, *n*,	108.365
And leſſer Fruſt. *x*, *p*, *æ*, *z*,	= Wine Gall. 41.861

Together = Wine Gallons 150.226, for the Capacity of the whole Spheroid, *prout ſuprà.*

There is an Elliptical Veſſel (Gauged in Probl. 12.) *the Rectangular Conjugate Diameters of each Baſe, being unequal,* viz. *the Conjugate Diameters aloft,* a, b, = 42 *Inches, and* m, p, = 35, *and the Conjugates below,* viz. c, d, = 39.1, *and* x, u, = 31.8, *and the Depth or Altitude* n, o, = 15 *Inches: To Gauge this Veſſel after a more brief and facile way, than is there given; And alſo to Inch the ſame, from the leſſer Baſe, (being here, from the bottom, upwards)* Fig. 1.

For Gauging of any such Elliptical Tunn, the Rule is,

1. Unto the Axe aloft, to wit, *a, b,* add the half of the Axe below, *viz.* of *c, d,* and multiply their Summ by *m, p,* (the lesser Diameter aloft) reserving that Product.

2. To the Axe below, *viz. c, d,* add the half of the Axe aloft, to wit, of *a, b,* and that Summ multiply by *x, u,* (the lesser Diameter below,) and this Product add unto that (above reserved) which being done, multiply their Summ by *n, o,* (the Vessels Depth); lastly, the Product (so found) mult. always by .0009283666+ if the Capacity be sought in Ale Gallons, but if in Barrels, *&c.* use the same Multiplicators given for a Conical Tunn, in *Probl.* 37.

a, b, = 42
½ of *c, d,* = 19.55

The Summ = 61.55
m, p, = 35

30775
18465

First Product = 2154.25

c, d, = 39.1
½ of *a, b,* = 21

Summ = 60.1
60.1

60.1
x. n. = 31.8

318
19080

Second Product = 1911.18
First Product = 2154.25

Their Summ = 4065.43
The Depth *n*, *o*, = 15

20327 15
406543

60981.45
Multiplicator for Ale Gall. .000928367+

42687015
36588870
18294435
48785160
12196290
54883305

Ale Gallons = 56.61316579215
The Veſſels whole Capacity.

The Rule for Inching ſuch Veſſels.

1. The Difference of the two Axes, *viz.* of *a*, *b*, and *c*, *d*, divided by the Depth, *n*, *o*, quotes (here .193333) which call *D*.

2. To the tripled leſſer Axe, *c*, *d*, add 1 ½ *D*, and that Summ multiply by the leſſer Diameter below, *viz* *x*, *u*; and that Product reſerve.

3. Unto 1 ½ of the leſſer Axe, *c*, *d*, add the number, *D*, noting the Summ; then the difference of the two leſſer Diameters; to wit, of *m*, *p*, and *x*, *u*, divided by the Depth, quotes (here .213333) which call *d*, whereby multiply the Summ above noted, and unto that Product add the reſerved Product, (which done) that Summ multiplied by .00092836 7+ produces the firſt Fruſtum in Ale Gallons.

4. To the leſſer Axe, to wit, *c*, *d*, tripled, add 4 ½ *D*, and that Summ multiply by the leſſer Diameter of the leaſt Baſe, *viz.* by *x*, *u*, and that Product reſerve; then to 4 ½ times, the leſſer Axe *c*, *d*, add *D* ſeptupled, and that Summ multiply by *d*, and the Product (ſo accruing) add unto the reſerved Product, which done, their Summ, multiplied by the aforeſaid Multiplicator, produces the firſt Difference in Ale Gallons.

5. To *c*, *d*, (the leſſer Axe) tripled, add 12 *D*, and that Summ multiply by *d*, reſerving the Product; then multiply *u*, *x*, the leſſer Diameter of the leaſt Baſe, by 3 *D*, and this Product add unto that reſerved; and the Summ (ſo accruing) multiplied by the ſaid Multiplicator, produces the ſecond Difference.

6. The ſextupled Rectangle of *D* into *d*, (multiplied by the common Multiplicator) produces the third Difference,

Operation

Operation.

$a, b, = 42$

$c, d, = 39.1$

Difference 2.9 (.193333 = *D*.
Depth 15

$m, p, = 35$

$x, u, = 31.8$

Difference 3.2 (.213333 = *d*
Depth 15

$c, d, = 39.1$
3

Tripled = 117.3
$1\frac{1}{2} D = .29$

The Summ 117.59
$u, x, = 31.8$

94072
11759
35277

First Product = 3739.362

$1\frac{1}{2}$ of $c, d, = 58.65$
$D = .193333$

The Summ $= 58.843333$
$d, = .213333$

Second Product $= 12.553$ &c.
First Product $= 3739.362$

The Summ $= 3751.915$
Multiplicator $.000928366$

First Frustum $=$ Ale Gall. 3.48316 &c.

$c, d,$ tripled $= 117.3$
$4\frac{1}{2} D = .87$

The Summ $= 118.17$
$u, x = 31.8$

First Product $= 3757.806$

$c, d, = 39.1$
4.5

$4\frac{1}{2} c, d, = 175.95$
$7 D = 1.3533$

The Summ $= 177.3033$
$d, = .213333$
Second Product $= 37.825$ &c.
First Product $= 3757.806$

The Summ $= 3795.631$
3795

3795.631
Multiplicator = .00092 8367-|-

Product, first Diff. = Ale Gall. 3.52375-|-*&c.*

c, *d*, tripled = 117.3
12 *D* = 2.32

The Summ = 119.62
d = .213333

First Product = 25.5189-|-

u, *x*, = 31.8
3 *D* = .58

Second Product 18.444
First Product 25.5189-|-

The Summ = 43.9629
Multiplicator = .00092 8367-|-

Product, Second Difference = .04081 *&c.*

6 Rectangles of *D* into *d*, = .247466-|-
The common Multiplicator .00092 8367-|-

Product = Ale Gallons .00023 0-|-
the third Difference.

We

Wet Inches.	*First Frustum, at 1 Inch in the Tunn, Ale Gallons.*	*First Differences.*	*Second Differences.*	*Third Differences.*
1	3.48316			
		3.52375	.04081	
2	7.00691			.00023
		3.56456	.04104	
3	10.57147			.00023
		3.60560	.04127	
4	14.17707			.00023
		3.64687	.04150	
5	17.82394			.00023
		3.68837	.04173	
6	21.51231			.00023
		3.73010	.04196	
7	25.24241			.00023
		3.77206	.04219	
8	29.01447			.00023
		3.81425	.04242	
9	32.82872			.00023
		3.85667	.04265	
10	36.68539			.00023
		3.89932	.04288	
11	40.58471			.00023
		3.94220	.04311	
12	44.52691			.00023
		3.98531	.04334	
13	48.51222			.00023
		4.02865	.04357	
14	52.54087			
		4.07222		
15	56.61309			

Fig. 15. *Vid. Probl.* 13. *pag.* 19.

There is a Vessel, Circular below, and Elliptical above, (Gauged in Probl. 13.*) the Diameter of the Circular Base,* c, d, *or* u, x, *is* 40 *Inches, the Elliptical Bases Axe* a, b, 45 *Inches, and the lesser Diameter* m, p, 43, *and the Depth* 30 *Inches; To Gauge this Vessel after a more facile and compendious manner, than is there laid down.*

For

For finding the whole liquid Capacity, of any such like Tunn,

The Rule is:

1. To the Diameter of the Circular Baſe *c, d,* or *u, x,* add the half of the Ellipſes leſſer Diameter, *viz.* of *m, p,* and of that Summ, multiplied by the Diameter of the Circular Baſe, reſerve the Product; then unto *m, p,* (the Ellipſes leſſer Diameter) add the Semidiameter of the Circular Baſe, *viz. c, o,* or *u, o,* (which done) that Summ multiplied by *a, b,* (the Ellipſes Axe) and the Product added unto the afore reſerved Product, the Summ ſo accruing multiplied by the Tunns Depth or Altitude, and that Product by the proper Multiplicator, given for a Conical Tunn (in *Probl.* 37.) this laſt Product is equal to the Tunns whole liquid Capacity, ſought.

c, d,	= 40
½ of *m, p,*	= 21.5
The Summ	= 61.5
c, d,	= 40
First Product	= 2460

m, p,

$m, p, = 43$
$c, o, = 20$

The Summ $= 63$
The Axe $a, b, = 45$

315
252

Second Product $= 2835$
First Product $= 2460$

The Summ $= 5295$
Tunns Depth $n, o, = 30$

Product $= 158850$
Multiplicator for Ale Gallons .000928366+

464183300
74269328
74269328
46418330
9283666

Ale Gallons $= 147.4710344100$
the Tunns whole Capacity.

Another INSTANCE.

Fig. 15. *Vid. Probl.* 13. *There is a Tunn, Elliptical aloft, and Circular below; the Axe of the Elliptical Base,* a,b, *equal to* 210; *and the lesser Diameter* m, p, *equal to* 140 *Inches; and the Diameter of the Circular Base,*

Base, viz. *either* c, d, *or* u, x, *equal to* 210, *and the Depth equal to* 24 *Inches: How many Ale Gallons is this Tunns whole Capacity, and how many, upon each Inch?*

First to find the Tunns whole Capacity by the Rule, above-given.

c, d, =	210
$\frac{1}{2}$ of *m, p,* =	70
Summ =	280
c, d, =	210
	2800
	56
First Product =	58800

m, p, =	140
c, o, =	105
Summ =	245
Axe *a, b,* =	210
	2450
	490
Second Product =	51450
First Product =	58800
Summ =	110250

110250
Depth *n*, *o*, = 24

441000
220500

Product = 2646000
Multiplicator = .000092 83666

55701996
37134664
55701996
18567332

Resp. 2456.458 0236000 Ale Gall.
The Tunns whole Capacity, from the Circular Base upwards.

The Rule for Inching this Tunn, is this, viz.

1. The Difference of *c*, *d*, (the Circular Bases Diameter) and *m*, *p*, (the Ellipses lesser Diameter) divided by *n*, *o*, (the Tunns Depth) quotes a number, which call *D*.

2. From the tripled Diameter of the Circular Base, subduct $1\frac{1}{2}$ *D*, and that Difference multiply by the Diameter of the Circular Base, *c*, *d*, or *u*, *x*; (which done) that Product multiplied by the Common Multiplicator (aforesaid) produces the first Frustums liquid Capacity.

3. From the tripled Diameter of the Circular Base, subduct $4\frac{1}{2}$ *D*, and that Difference mul-

multiply by the Diameter of the Circular Base, and that Product again by the said Common Multiplicator; which done, this last Product shews the first Difference.

4. The Diameter of the Circular Base, *viz.* *c*, *d*, or *u*, *x*, being multiplied by 3 *D*, aforesaid, and that Product, multiplied again by the Common Multiplicator, produces the second Difference.

The Operation.

c, *d*. = 210

m, *p*, = 140

Difference = 70 (*D* = 2.91667 +
n, *o*, 24

c, *d*, tripled = 630
$1\frac{1}{2}$ *D* = 4.375

Difference = 625.625
c, *d*, = 210

6256250
1251250

Product = 131381.250
Multiplicator for Ale Gall. = .000928367

Ale Gallons = 121.970027 &c.
is = first Frustums Capacity.

c, *d*,

c, d, tripled = 630

$4\frac{1}{2}$ D = 13.125

Difference = 616.875

$c\ d$ = 210

6168750

1233750

Product = 129543.75

Multiplicator for Ale Gall. .000928367

Ale Gallons = 129.26414 &c. the first Difference.

3 D = 8.75

c, d, = 210

8750

1750

Product = 1837.50

Multiplicator for Ale Gall. = .000928367

Ale Gallons = 1.7058744 &c. the second Difference.

Observe here, That (in this Case) the second Difference must be continually subducted from the first Difference, and the first Difference (so diminished) to be added unto the first Frustum, and so on, to every Inch of the Tunns depth; as appears in the ensuing Example.

Wet

Wet Inches	1. *Frustum in Inch. Ale Gal. in the Tunn.*	*First Differences.*	*Second Differences.*
1	121.970	120.264	1.706
2	242.234	118.558	1.706
3	360.792	116.852	1.706
4	477.644	115.146	1.706
5	592.790	113.440	1.706
6	706.230	111.734	1.706
7	817.964	110.028	1.706
8	927.992	108.322	1.706
9	1036.314	106.616	1.706
10	1142.930	104.910	1.706
11	1247.840	103.204	1.706
12	1351.044	101.498	1.706
13	1452.542	99.792	1.706
14	1552.334	98.086	1.706
15	1650.420	96.380	1.706
16	1746.800	94.674	1.706
17	1841.474	92.968	1.706
18	1934.442	91.262	1.706
19	2025.704	89.556	1.706
20	2115.260	87.850	1.706
21	2203.110	86.144	1.706
22	2289.254	84.438	1.706
23	2373.692	82.732	
24	2456.424		

LIV. Probl.

Fig. 4. *There is a Pyramidal Tunn, or the Fruſtum of a Pyramid, conſiſting of two parallel quadrangular Baſes, and equal ſides, at each Baſe: the four ſides aloft,* viz. c, a; a, d, *&c. are each* 200 *Inches; and* n, o; p, o, *&c. below, each* = 128; *and the Fruſtums Altitude, or Depth of the Tunn,* b, o, = 24 *Inches: How many Beer Barrels may this whole Tunn contain, and how many upon each Inch of the depth?*

The Rule for Gauging the whole Tunn, (as the Fruſtum of a Pyramid) may be this, viz.

1. Square the greater ſide, and that Square (being equal to the Area of the greater Baſe in Square Inches) inſert or ſet down; and in like manner find the Area of the other, or leſſer Baſe, and inſert it directly under the former Area.

2. Multiply into each other the ſide of the greater and leſſer Baſe given, and their Rectangle, or Product, inſert under the two Areas, or numbers aforefound, and their Summ multiplied by $\frac{1}{3}$ of the Tunns Depth, produces its ſolidity in Cubical Inches.

3. The Tunns ſolidity (ſo found in Cubical Inches) multiply (if the Capacity be deſired in Ale Gallons) by .0035461; if in Ale Barrels,

rels, by .000110815 3 ; and if in Beer Barrels, by .000098502 8 ; and the Product ſo accruing, is the Pyramidal Tunns Capacity, required.

The Area of the greater Baſe aloft = 40000
The Area of the leſſer Baſe below = 16384
The Rectangle of *c*, *a*, and *n*, *o*, the greater and leſſer ſides, = 25600

Their Summ = 81984
$\frac{1}{3}$ of the Tunns Depth = 8

Tunns ſolidity in Cubical Inches = 655872
Given Multiplicat. for Beer Bar. .000985028

5246976
1311744
32793600
5246976
5902848

The Tunns Capacity = 64.605228441 6
being 64 $\frac{1}{2}$ Beer Barrels, 3 Gallons, and 3 Quarts, *proximè*.

For Inching this Tunn, from the leſſer Baſe, to the greater, obſerve the Rule of *Probl.* 38. (*pag.* 77.) for Inching a Conical Tunn ; for, as this is a ſquare Pyramid, ſo a Cone may be ſaid to be a round Pyramid, and the Rule for Inching the Fruſtum of a Pyramid is the ſame (*mutatis mutandis*) with that for Inching the

Fruſtum of a Cone. Here note likewiſe, That inſtead of the Multiplicators, (there given for a Conical Fruſtum) you are to uſe theſe, for the Fruſtum of a Pyramid, *viz.*

The Multiplicators for	Ale Gallons,	.0011820334+
	Ale Barrels,	.0000369385 3+
	Beer Barrels,	.0000328245 6+

c, a, = 200

n, o, = 128
dep.

b, o, 24 : 72 :: 1 : 3, the common Addend.

1. The tripled Square of *n, o,* 128 = 49152
Tripled Rectangle of *n, o,* and Addend = 1152
The Square of the Addend = 9

The tripled Solidity in Cub. Inch. of the Tunn upon the firſt Inch, = 50313
Multiplicator for Beer Bar. = .000003282456

9847368
3282456
9847368
164122800

Beer Barrels = 1.651502087 28
is the Tunns Capacity upon the firſt Inch, from the leſſer Baſe.

2. The

2. The tripled Square of 128 = 49152
The nonocupled Rectangle of 128 and the Addend = 3456
The septupled Square of the Addend = 63

Summ = 52671
The Multiplicator = .000032 82456

3282456
22977192
19694736
6564912
164122 80

Beer Barrels = 1.72 8902 39976
is the Tunns Capacity upon the second Inch, from the lesser Base.

3. The sextupled Rectangle of 128, and the Addend = 2304
The dodecupled Square of the Addend = 108

Summ = 2412
The Multiplicator = .000032 82456

6564912
3282456
13129824
6564912

Of a Barrel .0791 7283872
the Difference between the Tunns Capacities upon the second and third Inch.

4. The sextupled Square of the Addend = 54
The Multiplicator = .00003282456

13129824
16412280

Of a Beer Barrel .00177252624

The third Diff. from whence (together with the former two) the Tunn is Inched, as follows.

Inches.	*The Tunns Capacity from Inch to Inch from the bottom upwards in Beer Bar.*	*Second Differences.*	*Third Differences.*
1	1.65150		
		.07740	
2	1.72890		.00177
		.07917	
3	1.80807		.00177
		.08094	
4	1.88901		.00177
		.08271	
5	1.97172		.00177
		.08448	
6	2.05620		.00177
		.08625	
7	2.14245		.00177
		.08802	
8	2.23047		.00177
		.08979	
9	2.32026		.00177
		.09156	
10	2.41182		.00177
		.09333	
11	2.50515		.00177
		.09510	
12	2.60025		.00177
		.09687	
13	2.69712		.00177
		.09864	
14	2.79576		.00177
		.10041	
15	2.89615		.00177
		.10218	
16	2.99835		.00177
		.10395	
17	3.10230		.00177
		.10572	
18	3.20802		

Sum 43.01350

Inch-

Inch.	*Tunns Capacity in Beer Bar.*	*Second Differences.*	*Third Differences*
			.00177
19	3.31551	.10749	.00177
20	3.42477	.10926	.00177
21	3.53580	.11103	.00177
22	3.64860	.11280	.00177
23	3.76317	.11457	.00177
24	3.87951	.11634	
Sum	21.56736		
	43.01350		

Total 64.58086 Beer Barels, being too little by 3 Quarts; which ſmall Error might have been removed, by taking the Fraction unto a greater number of places.

Now from this Table, by ſubduction only, is made the enſuing Table of this Tunn; ſhewing (by meer ocular inſpection) how many Beer Barrels and Gallons remain in the Tunn at any number of dry Inches, found by the Gauging Rod.

The Table conſiſts of four Columns; the firſt is Inches dry, the ſecond ſhews the number of Beer Barrels remaining in the Tunn at each dry Inch of the Depth, the third Column ſhews the reſpective Decimal Parts of a Barrel redundant, and in the fourth Column thoſe Decimal Parts are *(propè verum)* reduced into Gallons.

Memorandum, That the Drip of the Tunn muſt always be found and allowed for, according to the Rule in *Probl.* 40.

Inches

Inches dry.	Beer Barrels in the Tunn.	Decimal Parts redundant.	The Decimals, reduced into Gallons.
0	64	.5809	21
1	60	.7014	25
2	56	.9382	34
3	53	.2896	10 1/2
4	49	.7538	27
5	46	.3290	12
6	43	.0135	— 1/2
7	39	.8055	29
8	36	.7032	25 1/2
9	33	.7048	25 1/2
10	30	.8086	29
11	28	.0128	— 1/2
12	25	.3157	11
13	22	.7154	25 1/4
14	20	.2102	7 3/4
15	17	.7984	28 1/4
16	15	.4781	17
17	13	.2476	8 3/4
18	11	.1051	3 3/4
19	9	.0489	1 3/4
20	7	.0772	2 1/2
21	5	.1882	7
22	3	.3801	14
23	1	.6514	23 3/4
24	0	.0000	00

There

There is a Pyramidal Tunn, or Veſſel, whereof the Baſes are Rectangular Parallelograms, unequal; viz. c, a, *the length above*, = 300 *Inches*; a, d, *the breadth*, = 140: n, o, *the length below*, = 270; o, p *the breadth*, = 126: *and* b, o, *the depth*, = 20 *Inches: How many Ale Gallons may this Tunn contain; and how many, upon each Inch?* Fig. 4.

The Rule for Gauging the whole Tunn, Viz.

1. Unto the greater length, *c*, *a*, add the half of the leſſer length, *n*, *o*, and that Summ, multiplied by the greater breadth, *a*, *d*, call the firſt Product, which reſerve.

2. Unto the leſſer length, *n*, *o*, add the half of *c*, *a*, the greater length; and that Summ, multiplied by the leſſer breadth, *o*, *p*, call the ſecond Product.

3. The Summ of the aforeſaid two Products, multiply by the proper Multiplicator, (which the Queſtion requires) given upon the 54th *Probl. Pag.* 164. and that Product, again, by the Tunns depth, *b*, *o*, (which done) the Product, ſo found, is equal to the Tunns Capacity, required.

c, *a*,	= 300
$\frac{1}{2}$ of *n*, *o*,	= 135
Summ	= 435
a, *d*,	= 140
Firſt Product	= 60900

n, o,

$n, o, = 270$
$\frac{1}{2}$ of $c, a, = 150$

Summ $= 420$
$o, p, = 126$

Second Product $= 52920$
First Product $= 60900$

Summ $= 113820$
Multiplicator for Ale Gall. $= .001182033$

134.539 *&c.*
$b, o,$ Tunns depth $= 20$

Resp. Ale Gallons $= 2690.780$ *&c.* the Tunns whole Capacity; being 2690 Gallons, and 3 Quarts, *proximè.*

In Inching the Tunn, it is necessary to know the Common Addends, or Subducends, for constituting its increasing, or decreasing lengths and breadths, upon each Inch of the depth; which are thus found, *viz.*

The greater length $= 300$

The lesser length $= 270$

As the depth 20 to diff. 30 :: 1 : 1.5 the Common Addend to the lesser length, or Subducend from the greater, (signified in the Work by, *D.*)

The

The greater breadth = 140

The lesser breadth = 126

As the depth 20 to diff. 14 :: 1 : .7 (signified by, *d*, in the Work) the Common Addend to the lesser breadth, or Subducend from the greater, according as occasion requires.

To Inch this Tunn, from the lesser Base, to the greater, pursue the Direction of the Rule for Inching an Elliptical Vessel, given in *Pag.* 149. remembring to use the Multiplicators in *Pag.* 164.

Operation.

$3\,n, o, = 810$
$1\,\frac{1}{2}\,D = 2.25$

Summ = 812.25
$o, p, = 126$

First Product = 102343.50

$1\,\frac{1}{2}$ of $n, o, = 405$
$D = 1.5$

Summ = 406.5
$d = .7$

Second Product = 284.55
First Product = 102343.50

Summ = 102628.05

102628.05
Multiplicator for Ale Gall. = .001182033

121.30974 &c.

The first Frustum, at 1 Inch, from the lesser Base.

$3 n, o_1 = 810$
$4\frac{1}{2} D = 6.75$

Summ = 816.75
$o_1 p_2 = 126$

First Product = 102910.5

$4\frac{1}{2} n_1 o_1 = 1215$
$7 D = 10.5$

Summ = 1225.5
$d_1 = .7$

Second Product = 857.85
First Product = 102910.5

Summ 103768.35
Multiplicator = .001182033

Ale Gallons = 122.6576 &c.

the first difference.

$3 n_1 o_1 = 810$
$12 D_1 = 18$

Summ = 828

828

828
d, = .7

First Product = 579.6

v, p, = 126
3 D, = 4.5

Second Product = 567
First Product = 579.6

Summ = 1146.6
Multiplicator = .001182033

Ale Gallons = 1.35532 &c.
the second difference.

D, = 1.5
d, = .7

Rectangle of D, d, = 1.05
6

6 Rectangles = 6.30
Multiplicator = .001182033

Ale Gallons = .007447 &c.
the third difference.

Wet Inches.	*First Frustum at 1 Inch, in Ale Gallons.*	*First Differences.*	*Second Differ.*	*Third Differ.*
1	121.3097 —	122.6576 —		
2	243.9673		1.3553 —	
		124.0129		.0075 +
3	367.9802		1.3628	
		125.3757		.0075
4	493.3559		1.3703	
		126.7460		.0075
5	620.1019		1.3778	
		128.1238		.0075
6	748.2257		1.3853	
		129.5091		.0075
7	877.7348		1.3928	
		130.9019		.0075
8	1008.6367		1.4003	
		132.3022		.0075
9	1140.9389		1.4078	
		133.7100		.0075
10	1274.6489		1.4153	
		135.1253		.0075
11	1409.7742		1.4228	
		136.5481		.0055
12	1546.3223		1.4303	
		137.9584		.0075
13	1684.3007		1.4378	
		139.4162		.0075
14	1823.7169		1.4453	
		140.8615		.0075
15	1964.5784		1.4528	
		142.3143		.0075
16	2106.8927		1.4603	
		143.7746		.0075
17	2250.6673		1.4678	
		145.2424		.0075
18	2395.9097		1.4753	
		146.7177		.0075
19	2542.6274		1.4828	
		148.2005		
20	2690.8279			

Being 74 Beer Barrels, 26.75 Gall. which differs from the Calculation only .048 of a Gall. being nigh $\frac{1}{3}$ of a Pint, (an Error in these Cases not considerable) which might nevertheless (if need required) be with little more pains, taken away.

From hence (with much facility) may be raised a Table of the Beer, or Ale Barrels and Gallons, remaining in the Tunn, at any number of dry Inches, found by the Rod, according to that practical Method, in *Pag.* 168. THE

THE SECOND PART
TREATS OF
The ART of Wine, Brandy, &c.
CASK-GAUGING:
Explained in *xiiij.* PROBLEMS.

LV. PROBL.

A *Wine Pipes Boung Diameter* = 29 *Inches, and length* 48; *To find the Diagonal Arithmeticaly.*

The Square of 29, the Boung Diameter = 841
The Square of 24, the Semi-length = 576

The Square of the Diagonal = 1417

3,1513698 : 1417

1,5756849 : *Resp.* the Diagonal = 37.6431 +

LVI.

LVI. PROBL.

In the taking of Casks Dimensions, it often so happens, that you cannot come to take the Length of the Vessel, and yet you may take the Diameters at Head and Boung, with the Diagonal, whereby you may find the length.

EXAMPLE.

A Casks Boung Diameter is = 32, *and Diagonal found* = 38.5 *Inches; What is this Casks Length?*

The Square of the Diagonal 38.5 = 1482.25

The Square of 32 the Boung Diam. = 1024

The Square of the Semi-length = 458.25
viz. of 21.406. Inches.

So the Casks Length = 42.812

LVII. PROBL.

Fig. 12. *Having the Dimensions of a Spheroidal Tunn, or Cask; To find the Spheroids Axe, as* z, n.

The

The Rule.

Multiply the Square of the greater Semi-diameter, by the Square of the Tunns Depth, or of the Casks Semi-length; which done, that Product, divided by the Difference of the Squares of the greater and leſſer Semi-diameters given, quotes the Square of the Spheroids Semi-Axe; whereof the Square Root doubled, is = the Spheroids Axe required.

Example of a ſtanding Pipe.

Let d, u, w, d, *repreſent a Pipe, ſtanding on the Head, with its Diameters parallel to the Horizon; let* D, C, *or* C m, *the Semidiameter at the Boung, be* 14.5 *Inches; let* o, C, *or* C, i, *the Casks Semi-length, be* 24 *Inches; and* d, o, *or* o, u, *the Semi-diameter at the Head, be* 11.5 *Inches; To find* z, n, *the Spheroids Axe.*

C, i, = 24	d, o, = 11.5
24	11.5
96	575
48	115
576	115
	132.25

Square of $D, C, = 210.25$

Square of $d, o, = 132.25$

Difference of their Squares $= 78$

$D, C, 14.5$
14.5

725
580
145

210.25

q. of $C, i, = 576$

126150
147175
105125

$121104.00 : 5,0831584$

Diff. of Squares $78 = 1,8920946$

$3,1910638$

$1,5955319 : 39.40324+$

$39.40324+ = Z$, C, Spheroids Semi-axe.
2

$78.80648 = x, n$, the Spheroids Axe.

Cask-

Gask-Gauging is twofold, *viz.*
Either when the Cask is full, or when it is in part empty.

LVIII. Probl.

Of a Wine, Brandy, &c. Cask, having the Dimenſions; To find its whole Capacity in Wine, or in Ale Gallons, under theſe four Notions, viz. 1. as Spheroidal; 2. if Parabolical; 3. if Conical; 4. as Cylindrical.

EXAMPLE.

A Graves Hogſheads Boung Diameter is 25 *Inches, each Head Diameter* = 22, *and length* 32.0579 *Inches.*

1. *The Veſſel being aſſumed, as the middle Fruſtum of a Spheroid, intercepted between two Planes parallel, cutting the Axe at Right Angles: To find the Casks whole liquid Capacity.*

The Rule.

To the doubled Square of the Boung Diameter add the Square of the Head Diameter, and that Summ multiply by the length; (which done) that Product multiplied by .0011333331† produceth the Casks whole Capacity in Wine Gallons; but if the Capacity be ſought in Ale

Gallons, the Multiplicator is alwayes = .000928366+

25	22	32.0579
25	22	1734
125	44	1282316
50	44	961737
625	484	2244053
2		320579
1250		55588.3986
484		
1734		

55588.4+

Multiplicator for Wine Gallons .00113333

1667652
1667652
1667652
1667652
555884
555884

Respons. Wine Gallons = 63.000001372 the Graves Hogsheads Capacity, as Spheroidal.

Or the same may be done with more facility and expedition by the Cyclometrical Table.

2. *If*

2. *If a Cask be taken as the Middle Frustum of a Parabolical Spindle, intercepted between two Planes parallel, cutting the Axe at Right Angles; To find the Casks whole liquid Capacity.*

The Rule.

To the doubled Square of the Boung Diameter add the Square of the Head Diameter, and from that Summ subduct the quadrupled Square of the Difference of the Boung and Head Diameters; which done, that remainder multiplied by the Length, and that Product multiplied by either of the General Multiplicators (above mentioned for a Spheroidal Cask) produceth the Wine or Ale Gall. which the Cask may contain, as Parabolical, as in the aforesaid Hogshead.

Boung Diam. = 25	Head Diam. = 22
25	22
125	44
50	44
625	484
2	
1250	
484	
1734	

1734
36
1698

25
22
3
3
9
4
36

32.0579
1698
2564632
2885211
1923474
320579
54434.3142

54434.3142+
.00113333+
544343142
1633029426
1633029426
1633029426
1633029426
544343142
544343142

Wine Gallons = 61.692095746600 2 ;
the Hogsheads Capacity, as Parabolical.

3. If

3. *If a Cask be taken, as the middle Fruſtum of two Cones, abutting upon one common Baſe, intercepted between two Planes parallel, cutting the Axe at Right Angles; To find the whole liquid Capacity.*

The Rule.

To the Square of the Boung Diameter, add the Square of the Head Diameter, and from that Summ ſubduct the $\frac{1}{3}$ of the Square of the Difference of the Boung and Head Diameters; (which done) that Remainder multiply by 1.5 the length; and that Product being multiplied by either of the General Multiplicators aforegiven for a Spheroidal Cask, produces the Wine or Ale Gallons, which the Cask may contain, as Conical.

As for Inſtance, in the Hogſhead, aforeſaid.

25	22	25
25	22	—
—	—	22
125	44	—
50	44	3
—	—	3
625	484	—
484		9
—		Its $\frac{1}{3}$ = 3
1109		
—		
3		
—		
1106		

Length = 32.0579
Semi-length = 16.0289

1 ½ Length = 48.0868
1106

2885208
4808680
480868

53184.0008

.00113333 1+
53184—

4533324
9066648
1133331
3399993
5666655

Wine Gallons = 60.275075904
the Hogsheads Capacity, as Conical.

4. *If the Cask be Cylindrical; To find the whole liquid Capacity.*

Admit there be a Cylindrical Cask, whereof the Bases Diameter is = 23.5 Inches, and the length 32, to find the Casks whole Capacity in Wine Gallons, by aid of the Cyclometrical Table.

The

Wine Gall.

The Area of that Circle, whose Diameter is 23.5 Inch. *per* Tab. } = 1.8777

Casks length = 32

37554
56331

The Casks whole Capacity = 60.0864

II. To Gauge a Cask (partly empty) is various, according to the ſituation, or poſition of the ſame, that is, either when it ſtands upon the Head, with its Diameters parallel to the Horizon, or when it lies, with the Axe parallel to the Horizon.

LIX. Probl.

Having the Dimenſions of a Spheroidal Tunn, or ſtanding Cask, (partly empty) and Altitude of the Liquor; To find the Diameter of that Liquors ſuperficies.

For Example, take that of a ſtanding Pipe. Fig. 12.

Let d, u, w, d, *repreſent a Pipe, ſtanding on the Head, with its Diameters parallel to the Horizon; the Semi-diameter at the Bonng* D, C, *or* C, m, = 14.5 *Inches; the Casks Semi-length*

o, C,

o, C, *or* C, i, = 24 *Inches*; *the Semi diameter at the Head*, d, o, *or* o, u, = 11.5 *Inches*; *and the Altitude of Liquor, or wet Inches*, i, p, *or* C, p, = 36 *Inches*; (*being* .75 *of the Cask's length*) *To find the Quantity of* x, p, x, = *the Diameter of the Liquors ſuperficies, at the Altitude*, i, p, *or* C, p.

The Analogiſm.

As the Square of the Spheroids Semi-axe (firſt found by the Rule of *Probl.* 57.) is to the Square of the greater Semi-diameter; ſo is the Rectangle of the wet and dry Inches of the Spheroids Axe, to the Square of the Semidiameter of the ſuperficies of the Liquor in the Cask, whereof the Square Root doubled, is equal to the Diameter of the Liquors ſuperficies, at the Altitude of Liquor inquired of.

The Spheroids Semi-axe $C, n, = 39.40324$
$C, p, = 12$

Spheroids wet part $p, n, = 51.40324$
Its dry part $Z, p, = 27.40324$

20561296
10280648
15420972
205612960
35982268
10280648

1408.615322497

eo.ar. 6,8089362 :	As the Square of *Z*, *C*, the Spheroids Semi-axe ;
2,3227360 :	to the q. of *D*,*C*, the Semi-diameter Boung Circle ;
3,1487923 :	So 1408.6153—, ▭ of *n*, *p*, and *p*, *z* ;
2,2804645 :	to the Square of *x*, *p*, or *p*, *a*.
1,1402322— :	13.81123 = *x*,*p*,or *p*, *a*.
	2
	27.62246 = *x*, *p*, *a* ;

the Diameter of the Liquors superficies, when 36 Inches of the Pipes length is wet, *p*, *i*, being $\frac{3}{4}$ of the whole length *o*, *i*.

LX. Probl.

Having the Dimensions and liquid Capacity of a Spheroidal Tunn, or of a Cask, standing on the Head, (partly full and in part empty) with its Diameters parallel to the Horizon, and the Diameter of the Liquors superficies; To find what Quantity of Liquor remains therein, at any number of wet Inches proposed. Fig. 12.

The Graves Hogshead d, u, w, d, *its liquid Capacity, being* 63 *Wine Gallons, Boung Diameter,* D, m, 25 *Inches, length* o, i, 32.0579 : *Now* $\frac{1}{4}$ *of the Casks length,* viz. o,p, 8.014475 *Inches,*

Inches, being dry; how many Gallons remain in the Cask, the Diameter of the Liquors superficies being 24.28477 *Inches.*

The Rule.

1. To the doubled Square of the greater Diameter, add the Square of the lesser Diameter or of the Liquors superficies.

2. That Summ multiply by *C, p,* the wet Inches, above the Casks the Mediety.

3. That Product multiplied alwayes by .0011333331+ for Wine Gallons, or by .0009283666+ for Ale Gallons, produces the Wine or Ale Gallons in the Casks Frustums, *x, e, m, D,* that is, how much it is above half full; whereunto the Casks Semi-capacity being added, that Summ is the Answer of the Question.

The doubled q. of Diam. 25 is = 1250
q. of Diam. *x, e,* 24.28477 = 589.75—

1839.75—
C, p, wet Inches above Casks Mediety 8.0145

919875
735900
183975
14718000

14744.676375

Mul-

14744.676375
Multiplicator for Wine Gall. = .0011333

44234029125
44234029125
44234029125
14744676375
14744676375

The Capacity of the Casks Fruſtum, *x*, *e*, *m*, *D*, Win. Gall. } 16.710541735 7875
Casks Semicapacity 31.5

Reſp. Wine Gallons 48.21054 *&c.* remaining in the Tunn.

Wine Gall.
16.71054 Capacity of *x*, *e*, *m*, *D*.
2
33.42108 its doubled Capacity.
63 Casks whole Capacity.
29.57892
14.78946 Capacity of *d*, *u*, *e*, *x*.

Another way by the Cyclometrical Table.

Wine Gall.
$\frac{2}{3}$ of Area of Boung Circle, *D*, *m*, Diam. 25 Inches, } = 1.416663
$\frac{1}{3}$ of Area of leſſer Circle, Diameter being 24.28477 } = 0.668382
Area of the Mean Circle is Wine Gallons ——— } = 2.085045

C, *p*,

2.085045—

C, *p*, wet Inch. above Casks Mediety 8.01448+

16680360
8340180
8340180
2085045
166803600

16.71055145160

Casks Semi-capacity = 31.5

Resp. Wine Gallons = 48.2105 *prout supra.*
in the Cask.

Another way by the Table.

25 Inches Diameter doubled = 4.249989
24.28477 = 2.005146

Tripled Area of Mean Circle = 6.255135
$\frac{1}{3}$ of *C*, *p*, 8.01 *&c.* = 2.67149—

56296215
25020540
6255135
43785945
37530810
12510270

16.71053060115

A

A round Tunn, as D, m, w, d, *is of* 144 *Inches Diameter at the top, and* 108 *Diameter at the bottom, and* C, i, *the Depth,* 60 *Inches; and being, (by reason of the Curvity of the Sides,) taken as the second Frustum or Segment of a Spheroid: How many Ale Gallons is this Tunns whole Capacity; and how many Gallons remain in it, when* i, x, *the wet Inches are only* 50?

	Ale Gall.
$\frac{1}{3}$ of Area of the Circle at the top, Diam. *D, m*, being 144 Inches, is	= 38.5012,2054
$\frac{1}{3}$ of Area of the Circle at the bottom, Diam. *d, w*, being 108 Inch.	= 10.8284,6827
Area of the Mean Circle	= 49.3296,8881
The Tunns Depth	= 60
Tunns whole Capacity Ale Gallons.	= 2959.7813,2860

The Spheroids Axe z, n, *is found by the Rule in* Probl. 57. = 181.42296; *and* z, x, *the Spheroids dry part,* 100.71148; *and the Diameter of the Liquors superficies at* x, *is found* 143.12232.

Then

Then by the Cyclometrical Table.

	Ale Gall.
$\frac{1}{3}$ of Area of the Circle at the Diameter D, m, 144 Inches	= 38.5012,2054
$\frac{1}{3}$ of Area of Circle at ſuperficies of Liquor x, 143.12232 Inch.	= 19.0166,6189
Area of the Mean Circle	= 57.5178,8243
C, x, the Tunns dry Inches	= 10
The Tunn wants to fill it up, of Ale Gallons	= 575.1788,2430
Reſp. Remaining in the Tunn	2384.6025,043

There is Pipe, ſtanding on the Head, whoſe whole Capacity is 120.278 *Wine Gallons, Bouns Diameter* 29 *Inches, length* 48 *Inches: Now if* $\frac{1}{4}$ *of the Casks length*, viz. 12 *Inches be dry, and the Diameter of the Liquors ſuperficies,* x, p, x, *be* 27.62246 *Inches: How many Wine Gallons remain in this Pipe?*

29
29

261
58

841

841	27.62246
2	27.62246
1682	16573476
	11048984
	5524492
	5524492
	16573476
	19335722
	5524492
	763.000296451 6

The doubled Square of the Boung Diameter 29 Inches ——— = 1682

The Square of the leſſer Diameter, *x*, *p*, *a*, 27.62246 ——— = 763

Summ = 2445

C, *p*, the wet Inches above the Casks Mediety ——— = 12

Product = 29340

Multiplicat. for Wine Gall. = .0011333331—

2934

	4533324
	3399993
	10199979
	2266662

The Capacity of the Casks Frustum *x*, *p*, *a*, *m*, *D*, = 33.25193154 Wine Gallons.

33.25193
The Casks Semi-capacity = 60.13905

Resp. Wine Gallons = 93.39098
remaining in the Pipe.

Another way by the Cyclometrical Table.

Wine Gall.

$\frac{2}{3}$ of the Area of the Boung Circle, D, m, the Diameter being 29 Inches, = 1.90626

$\frac{1}{3}$ of the Area of the Circle, the Diameter x, p, a, being 27.62246 = 0.86474

Area of the Mean Circle in Wine Gallons = 2.77100

C, p, the wet Inches above the Pipes Mediety = 12

The Capacity of the Pipes Fruſtum, x, p, a, m, D, = 33.25200

The Pipes Semi-capacity = 60.139—

Resp. Remaining in Pipe *prout ſupra* 93.391
Wine Gallons.

LXI. Probl.

Having the Dimenſions and liquid Capacity of a Spheroidal Tunn, or ſtanding Cask, (in part full and partly empty) together with the quantity of remaining Liquor, and the Diameters of that Liquors ſuperficies; To find how many Inches of the Tunns depth, or of the Casks length, are wet. Fig. 12.

The Rule.

1. In a Tunn, the Difference between the propoſed remaining Liquor and the whole Capacity, is equal to the Tunns Fruſtum, D,C,m,x; that is the Tunns vacuity.

In a ſtanding Cask, the Difference between the propoſed remaining Liquor, and the Casks Semi-capacity, is equal to the Casks Fruſtum, either above, or below the Mediety.

2. This Difference, or Remainder, (be it of Tunn or Cask) if the Capacity be given in Wine Gallons, multiply always by 882.3551, and if in Ale Gallons, multiply always by 1077.161; but if the Capacity be propoſed in Ale Barrels, multiply the aforeſaid Difference always by 34469.143; and if in Beer Barrels, multiply always by 38777.778, and reſerve the Product.

3. Square the greater Diameter of the Tunn,

or ſtanding Cask, and unto the double of its Square, add the Square of the Diameter of the Liquors ſuperficies, which let be your Diviſor, or by direction of *Probl.* 63. make its correſpondent Multiplicator.

4. That reſerved Product, being divided by the Summ laſt mentioned, or multiplied by its correſpondent Multiplicator, quotes, or produces the Tunns dry Inches, *C, x,* from which, (the depth being known) the wet Inches are alſo made maniteſt; but if it be a Cask, the Quotient or Product is equal to the wet Inches above, or, to the dry Inches below the Casks Mediety; from which, the Anſwer of the Queſtion ariſes.

To perform this another way, the Rule is this, viz.

As the Square of the greater Diameter of a Spheroidal Tunn, or ſtanding Cask, *D, C, m,* is to the Square of *C, n,* or *C, z,* the Spheroids Semi-axe: So is the Square of the Diameter of the Liquors ſuperficies; to the Rectangle of the wet and dry Inches of the Spheroids Axe, *n, p,* and *z, p:* Which ſubducted from the Square of *n, C,* the Spheroids Semi-axe; the ſquare Root of that Remainder is equal to the wet Inches above the Casks Mediety, as *C, p;* or to the dry Inches below the ſame, as *C, x:* From which, the Tunns, or Casks, wet and dry Inches cannot be unknown.

To

To perform this by the Cyclometrical Table.

In a Tunn, the Difference of its whole Capacity and the remaining Liquor, is always equal to *D, C, m, x*, the Tunns vacuity.

In a ſtanding Cask, the Difference of its Semi-capacity and the remaining Liquor, is always equal to the Capacity of the Casks Fruſtum, above the Mediety; as, *x,p,æ,m,C,D*; or unto the vacant Fruſtum, below the Mediety, as, *D, C, m, x*.

(Which obſerved) This Vacuity or Capacity, ſo found, being divided by the Area of the Mean Circle; *(i. e.)* the Summ of two thirds of the Area of the greater Circle, and one third of the Circle at the Liquors ſuperficies, or multiplied by the correſpondent Multiplicator thereunto; That Quotient, or Product, is always equal to *C, x*, the Tunns dry Inches; and in a Cask, unto *C, p*, the wet Inches above the Mediety; or unto *C, x*, the dry Inches below the Mediety.

As for EXAMPLE.

There is a Spheroidal Tunn, as, D, C, m, w, i, d, *whoſe depth is* 60 *Inches, upper Diameter,* D, C, m, 144, *whole Capacity* 2959.78133 *Ale Gallons; and it being known, that there remains in the Tunn, only* 2384.60251 *ſuch Gallons, and that the Diameter of the Liquors ſu-* Fig. 12.

perficies is 143.12232; *the Question is, how many the wet Inches are?*

The FIRST WAY.

Greater Diameter 144

144

576
576
144

20736
2

41472
20483.9985

61955.9985

143.12232
143.12232

20483.9985

Ale Gallons.

The Tunns whole Capacity = 2959.78133

Remaining Liquor = 2384.60251

Tunns Vacuity = 575.17882
1077.161

57517882
345107292
57517882
402625174
402625174
575178820

619560.19393002

619560.192—

The correspondent Multiplicat. for 61955.9985 } .00001614049+

5576041728
2478240768
2478240768
619560192
3717361152
619560192

Tunns dry Inch. *C, x*, 10,000005083374̇08

Tunns depth, *C, i*, = 60 Inches.

Resp. wet Inch. *x, i*, 50

The SECOND WAY.

co. ar. 5,6832750 : As the Square of 144, the greater Diam. *D, C, m*;

3,9153245 : to 8228.5726 the Square of 90.71148, the Spheroids Semi-axe, *C n*:

4,3114140 : So is the q. of 143.12232, the Diameter of the Liquors superficies, at *x*;

3,9100135 : to 8128.5726, the Rectangle of *n, x*, and *x, z*, the Spheroids Axes wet and dry parts.

8128.5726

100 the Square of 10 Inch. for C, x, dry Inch

which ſubducted from 60 the Tunns depth, C, i.

Reſp. Tuns wet Inches 50 for x, i.

The THIRD WAY, *by the Cyclometrical Table.*

	Ale Gall.
$\frac{2}{3}$ of Circles Area, Diam. 144 =	38.50122
$\frac{1}{3}$ of Area Circles Diameter =	19.01666
	57.51788
	Ale Gall.
The Tunns whole Capacity =	2959.7813
Remaining Liquor	2384.6025
The Tunns Vacuity =	575.1788
Correſpondent Mult. for Area Mean Circle, 57.51788 } =	.0173859

51766092
28758940
46014304
17255364
40262516
5751788

Tunns dry Inches, C, x, = 10.000001098 92

Tunns depth = 60

Reſp. Tuns wet Inch. ſought 50

There

There is a Wine Hogshead, standing on the Head, as d, u, w, d, *whose Bounng Diameter* D,m, *is* = 25 *Inches, the length* o, i, = 32.0579 *Inches, and whole Capacity* = 63 *Wine Gall. Now if the Liquor remaining in the Cask, be only* 48.21054 *such Gallons, and the Diameter of the Liquors superficies, be* 24.28477 *Inches; how many Inches of the Casks length are wet?* Fig. 12.

$\frac{2}{3}$ of 25 *per* Table = 1.416663
$\frac{1}{3}$ of 24.28477 = 0.668382

2.085045

Wine Gall.
Remaining in the Cask = 48.21054

The Casks Semi-capacity = 31.5

16.71054

Correspondent Multipl. to Area of Mean Circle, 2.085045 = .479606

10026324
100263240
15039486
11697378
6684216

C, p, the Casks wet Inches above the Mediety, 8.014475247 24

C, i,

8.01447

C, i, the Casks Semi-length = 16.02895

Resp. p, i, the Casks wet Inches being $\frac{3}{4}$ of the Length, = 24.04342

Observe here, That if the ſtanding Cask, laſt mentioned, with the Liquor in it, (which wets $\frac{3}{4}$ of its length) be laid down with its Axe parallel to the Horizon, the Boung Diameters wet part will be but 17.8791 Inches; wanting .8709 of an Inch of $\frac{3}{4}$ of its Boung Diameter, as may be eaſily proved.

There is a Wine Pipe, ſtanding on the Head, as d, u, w, d, *in* Fig. 12. *whoſe Boung Diameter* D, m, *let be* 29 *Inches*, 0, i, *the length* 42 *Inches, whole Capacity* 120.278 *Wine Gallons: Now if there remain in this Cask, only* 93.39098 *ſuch Gallons, filling the ſame up unto* x, p, æ; *and the Diameter of the Liquors ſuperficies*, x, p, æ, *be* 27.62246; *how many Inches are* p, i, *that is, how many Inches of the Casks length are wet?*

29
29

261
58

841
2

1682

27.62246
27.62246

16573476
11048984
5524492
5524492
16573476
19335722
5524492

763.00029645 16

The double q. of $D, m, =$ 1682

2445

Wine Gall.

Remaining Liquor = 93.39098

Pipes Semi-capacity = 60.13905

Capacity of $x, p, e, m, D, =$ 33.25193
Multiplicator for Wine Gall. 882.3551+

3325193
16625965
16625965
9975579
6650386
26601544
26601544

29340.010020343

Mul-

Multiplicator for 2445 = .000408998+

29340

16359920
1226994
3680982
817996

p, C, the Fruſtums Altitude above the Mediety, Inches } 12.000001320
C, i, the Casks Semi-length 24 Inches.

Reſp. p, i, Casks wet Inches, 36

Another way, by the Cyclometrical Table.

Wine Gall.

Remaining Liquor in the Pipe = 93.39098
The Pipes Semi-capacity = 60.13905
Capacity of *x, p, m, D*, the Pipes Fruſtum = 33.25193
Multiplicator for 2.771 the Area of the Mean Circle, is = .360881

3325193
26601544
26601544
19951158o
9975579

p, C, Inches = 11.99998975033
C, i, = 24

Reſp. p, i, = 36, *prout ſuprà.*

To

To Gauge a Cask, lying with the Axe pa-to the Horizon, (partly empty) is alſo two-fold, to wit, either when the ſuperficies of the remaining Liquor, cuts not the Casks Heads; or, when it doth cut the ſame.

LXII. Probl.

Of any Wine, Brandy, &c. Cask, (in part full and partly empty) lying with the Axe parallel to the Horizon, having the Dimenſions, and the Boung Diameters dry part, (i. e.) *the Casks vacant Fruſtums Altitude; To find the vacuity or wants, in Wine, or in Ale Gallons, when the ſuperficies of the remaining Liquor cuts not the Casks Head; known always by the Semidifference of the Boung and Head Diameters exceeding the number of dry Inches, or equality thereunto.*

The Rule.

1. The Spheroids Axe of the Cask divide by the Boung Diameter, or multiply it by the proper Multiplicator for the Boung Diameter.

2. That Quotient or Product multiply always for Wine Gallons by .004533343—, and for Ale Gallons always by .0037134671/2, reſerving the Product.

3. Mul-

3. Unto the wet Inches given, add the known Semi-diameter at the Boung, and by that Summ multiply the Square of the proposed dry Inches, noting that Product.

4. Multiply the reserved Product by the Product last mentioned; which done, the Product (so accruing), is equal to the Vacuity, or Wants of the Cask in Wine, or Ale Gallons, according to the purport of the Question.

EXAMPLE.

Fig. 13. *Let* d, u, w, d, *represent a Wine Cask, whereof let* D, m, *the Boung Diameter be* 25 *Inches, and the Spheroids Axe (found by the Rule in* Probl. 57.) 67.49402 *Inches, and let* D, y, *the dry part of the Boung Diameter be* 1.5 *Inch: How many Wine Gallons are wanting to fill up the Cask.*

1.5	*y, m,* the wet Inches =	23.5
1.5	*C, m,* Semi-diameter Boung =	12.5
75		36
15		
2.25		
36		
1350		
675		
81.00		

The

The Spheroids Axe = 67.49402+
Multiplicator for 25 = .04

2.6997608
General Mult. for Wine Gall. .00453343—

80992824
107990432
80992824
80992824
80992824
134988040
107990432

See another way, with out the Spheroids Axe, Probl. 32.

.0122389(417243544
81

122389
979112

Resp. The Cask wants .9913509 to fill it up, (being nigh a Wine Gallon) when the Liquor reaches to the upper Orbs of the Heads, known always by the equality of the dry Inches to the Semi-difference of the Boung and Head Diameters.

In the Cask aforesaid, supposing D, x, *the Boung Diameters dry part, be onely* $\frac{1}{2}$ *an Inch: How much of a Wine Gallon is the Vacuity?* Fig. 13.

.5
.5

.25 x, m,

X, m, the wet Inches = 24.5
C, m, Semi-diameter at the Boung = 12.5

37
.35
185
74
9.25

.0122389
9.25
611945
244778
1101501

Resp. .11320982 5 being nigh a Pint.

In the same Cask, supposing D, x, *the Boung Diameters dry part, be* $\frac{1}{4}$ *of an Inch: How much of a Wine Gallon is the Vacuity?*

.25
.25
125
50
.0625

X, m,

X, *m*, the wet Inches = 24.75
C, *m*, the Semidiameters Boung = 12.5

37.25
.0625

18625
7450
22350

2.328125

.0122389
2.328125

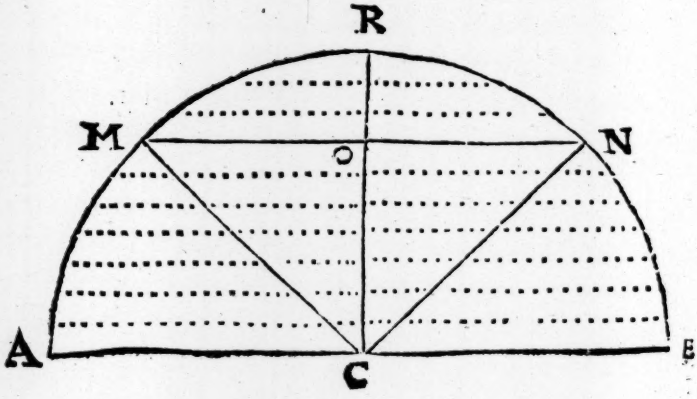
611945
244778
122389
979112
244778
367167
244778

.028493689 90625

The Wants are nigh $\frac{1}{4}$ of a Pint.

A TABLE OF THE AREA'S of SEGMENTS OF SUCH A *CIRCLE*,

Whereof the whole *Area* is 2.00000; the *Radius* being ſuppoſed to be divided by the Chord Lines, in the *Ratio* of 10000 parts, Arithmeticaly increaſing, and the *Area's* Calculated to the $\frac{1}{100000}$ part of a ſquare Inch; of great Uſe in *Geometry*, and *Gauging of Cask, in part empty, &c.*



Arrow.	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.0001		.9999		
.0010	.00004	.9990	.99996	.00011
.0020	.00015	.9980	.99985	.00013
.0030	.00028	.9970	.99972	.00015
.0040	.00043	.9960	.99957	.00017
.0050	.00060	.9950	.99940	.00019
.0060	.00079	.9940	.99921	.00021
.0070	.00100	.9930	.99900	.00022
.0080	.00122	.9920	.99878	.00023
.0090	.00145	.9910	.99855	.00024
.0100	.00169	.9900	.99831	.00026
.0110	.00195	.9890	.99805	.00027
.0120	.00222	.9880	.99778	.00028
.0130	.00250	.9870	.99750	.00029
.0140	.00279	.9860	.99721	.00031
.0150	.00310	.9850	.99690	.00032
.0160	.00342	.9840	.99658	.00032
.0170	.00374	.9830	.99626	.00034
.0180	.00408	.9820	.99592	.00034
.0190	.00442	.9810	.99558	.00035
.0200	.00477	.9800	.99523	.00036
.0210	.00513	.9790	.99487	.00037
.0220	.00550	.9780	.99450	.00038
.0230	.00588	.9770	.99412	.00039
.0240	.00627	.9760	.99373	.00039
.0250	.00666	.9750	.99334	

Arrow.	Area of Segmẽt.	Arrow	Area of Segmẽt.	Common Differences.
.0260	.00706	.9740	.99294	.00040
.0270	.00747	.9730	.99253	.00041
.0280	.00789	.9720	.99211	.00042
.0290	.00831	.9710	.99169	.00042
.0300	.00874	.9700	.99126	.00043
				.00044
.0310	.00918	.9690	.99082	.00045
.0320	.00963	.9680	.99037	.00046
.0330	.01009	.9670	.98991	.00046
.0340	.01055	.9660	.98945	.00046
.0350	.01101	.9650	.98899	.00047
.0360	.01148	.9640	.98852	.00047
.0370	.01195	.9630	.98805	.00048
.0380	.01243	.9620	.98757	.00048
.0390	.01291	.9610	.98709	.00049
.0400	.01340	.9600	.98660	.00050
.0410	.01390	.9590	.98610	.00051
.0420	.01441	.9580	.98559	.00052
.0430	.01493	.9570	.98507	.00053
.0440	.01546	.9560	.98454	.00053
.0450	.01599	.9550	.98401	.00053
.0460	.01652	.9540	.98348	.00054
.0470	.01706	.9530	.98294	.00054
.0480	.01760	.9520	.98240	.00055
.0490	.01815	.9510	.98185	.00055
.0500	.01870	.9500	.98130	.00056

Arrow	Area of Segmẽt.	Arrow	Area of Segmẽt.	Common Differences.
.0510	.01926	.9490	.98074	.00056
.0520	.01982	.9480	.98018	.00057
.0530	.02039	.9470	.97961	.00057
.0540	.02096	.9460	.97904	.00057
.0550	.02153	.9450	.97847	.00058
.0560	.02211	.9440	.97789	.00059
.0570	.02270	.9430	.97730	.00059
.0580	.02329	.9420	.97671	.00060
.0590	.02389	.9410	.97611	.00061
.0600	.02450	.9400	.97550	.00060
.0610	.02510	.9390	.97490	.00061
.0620	.02571	.9380	.97429	.00062
.0630	.02633	.9370	.97367	.00062
.0640	.02695	.9360	.97305	.00063
.0650	.02758	.9350	.97242	.00063
.0660	.02821	.9340	.97179	.00063
.0670	.02884	.9330	.97116	.00064
.0680	.02948	.9320	.97052	.00064
.0690	.03012	.9310	.96988	.00065
.0700	.03077	.9300	.96923	.00065
.0710	.03142	.9290	.96858	.00066
.0720	.03208	.9280	.96792	.00066
.0730	.03274	.9270	.96726	.00066
.0740	.03340	.9260	.96660	.00067
.0750	.03407	.9250	.96593	.00068

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.0760	.03475	.9240	.96525	.00067
.0770	.03542	.9230	.96458	.00068
.0780	.03610	.9220	.96390	.00069
.0790	.03679	.9210	.96321	.00069
.0800	.03748	.9200	.96252	.00069
.0810	.03817	.9190	.96183	.00070
.0820	.03887	.9180	.96113	.00070
.0830	.03957	.9170	.96043	.00070
.0840	.04027	.9160	.95973	.00071
.0850	.04098	.9150	.95902	.00071
.0860	.04169	.9140	.95831	.00072
.0870	.04241	.9130	.95759	.00072
.0880	.04313	.9120	.95687	.00072
.0890	.04385	.9110	.95615	.00073
.0900	.04458	.9100	.95542	.00073
.0910	.04531	.9090	.95469	.00073
.0920	.04604	.9080	.95396	.00074
.0930	.04678	.9070	.95322	.00074
.0940	.04752	.9060	.95248	.00075
.0950	.04827	.9050	.95173	.00074
.0960	.04901	.9040	.95099	.00076
.0970	.04977	.9030	.95023	.00075
.0980	.05052	.9020	.94948	.00076
.0990	.05128	.9010	.94872	.00076
.1000	.05204	.9000	.94796	.00077

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.1010	.05281	.8990	.94719	.00077
.1020	.05358	.8980	.94642	.00077
.1030	.05435	.8970	.94565	.00078
.1040	.05513	.8960	.94487	.00078
.1050	.05591	.8950	.94409	.00078
.1060	.05669	.8940	.94331	.00078
.1070	.05747	.8930	.94253	.00079
.1080	.05826	.8920	.94174	.00079
.1090	.05905	.8910	.94095	.00080
.1100	.05985	.8900	.94015	.00080
.1110	.06065	.8890	.93935	.00080
.1120	.06145	.8880	.93855	.00081
.1130	.06226	.8870	.93774	.00081
.1140	.06307	.8860	.93693	.00081
.1150	.06388	.8850	.93612	.00081
.1160	.06469	.8840	.93531	.00082
.1170	.06551	.8830	.93449	.00082
.1180	.06633	.8820	.93367	.00082
.1190	.06715	.8810	.93285	.00082
.1200	.06797	.8800	.93203	.00083
.1210	.06880	.8790	.93120	.00083
.1220	.06963	.8780	.93037	.00083
.1230	.07046	.8770	.92954	.00084
.1240	.07130	.8760	.92870	.00084
.1250	.07214	.8750	.92786	.00085

Arrow	Area of Segmēt.	Arrow	Area of Segmēt.	Common Differences
.1260	.07299	.8740	.92701	
.1270	.07384	.8730	.92616	.00085
.1280	.07469	.8720	.92531	.00085
.1290	.07554	.8710	.92446	.00085
.1300	.07639	.8700	.92361	.00085
				.00086
.1310	.07725	.8690	.92275	
.1320	.07811	.8680	.92189	.00086
.1330	.07897	.8670	.92103	.00086
.1340	.07984	.8660	.92016	.00087
.1350	.08071	.8650	.91929	.00087
				.00087
.1360	.08158	.8640	.91842	
.1370	.08246	.8630	.91754	.00088
.1380	.08333	.8620	.91667	.00087
.1390	.08421	.8610	.91579	.00088
.1400	.08509	.8600	.91491	.00088
				.00089
.1410	.08598	.8590	.91402	
.1420	.08687	.8580	.91313	.00089
.1430	.08776	.8570	.91224	.00089
.1440	.08865	.8560	.91135	.00089
.1450	.08955	.8550	.91045	.00090
				.00090
.1460	.09045	.8540	.90955	
.1470	.09135	.8530	.90865	.00090
.1480	.09225	.8520	.90775	.00090
.1490	.09315	.8510	.90685	.00090
.1500	.09406	.8500	.90594	.00091
				.00091

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.1510	.09497	.8490	.90503	.00091
.1520	.09588	.8480	.90412	.00091
.1530	.09679	.8470	.90321	.00092
.1540	.09771	.8460	.90229	.00092
.1550	.09863	.8450	.90137	.00092
.1560	.09955	.8440	.90045	.00093
.1570	.10048	.8430	.89952	.00093
.1580	.10141	.8420	.89859	.00093
.1590	.10234	.8410	.89766	.00094
.1600	.10328	.8400	.89672	.00094
.1610	.10422	.8390	.89578	.00094
.1620	.10516	.8380	.89484	.00094
.1630	.10610	.8370	.89390	.00094
.1640	.10704	.8360	.89296	.00095
.1650	.10799	.8350	.89201	.00095
.1660	.10894	.8340	.89106	.00095
.1670	.10989	.8330	.89011	.00095
.1680	.11084	.8320	.88916	.00095
.1690	.11179	.8310	.88821	.00096
.1700	.11275	.8300	.88725	.00096
.1710	.11371	.8290	.88629	.00096
.1720	.11467	.8280	.88533	.00096
.1730	.11563	.8270	.88437	.00096
.1740	.11659	.8260	.88341	.00096
.1750	.11755	.8250	.88245	.00097

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.1760	.11852	.8240	.88148	
.1770	.11949	.8230	.88051	.00097
.1780	.12046	.8220	.87954	.00097
.1790	.12143	.8210	.87857	.00097
.1800	.12240	.8200	.87760	.00097
				.00098
.1810	.12338	.8190	.87662	
.1820	.12436	.8180	.87564	.00098
.1830	.12534	.8170	.87466	.00098
.1840	.12633	.8160	.87367	.00099
.1850	.12732	.8150	.87268	.00099
				.00099
.1860	.12831	.8140	.87169	
.1870	.12930	.8130	.87070	.00099
.1880	.13030	.8120	.86970	.00100
.1890	.13129	.8110	.86871	.00099
.1900	.13229	.8100	.86771	.00100
				.00100
.1910	.13329	.8090	.86671	
.1920	.13429	.8080	.86571	.00100
.1930	.13529	.8070	.86471	.00100
.1940	.13630	.8060	.86370	.00101
.1950	.13731	.8050	.86269	.00101
				.00101
.1960	.13832	.8040	.86168	
.1970	.13933	.8030	.86067	.00101
.1980	.14034	.8020	.85966	.00101
.1990	.14136	.8010	.85864	.00102
.2000	.14238	.8000	.85762	.00102
				.00102

Arrow	Area of Segmẽt.	Arrow	Area of Segmẽt.	Common Differences.
.2010	.14340	.7990	.85660	.00102
.2020	.14442	.7980	.85558	.00102
.2030	.14544	.7970	.85456	.00103
.2040	.14647	.7960	.85353	.00103
.2050	.14750	.7950	.85250	.00103
.2060	.14853	.7940	.85147	.00103
.2070	.14956	.7930	.85044	.00103
.2080	.15059	.7920	.84941	.00103
,2090	.15162	.7910	.84838	.00103
.2100	.15265	.7900	.84735	.00104
.2110	.15369	.7890	.84631	.00104
.2120	.15473	.7880	.84527	.00104
.2130	.15577	.7870	.84423	.00104
.2140	.15681	.7860	.84319	.00104
.2150	.15785	.7850	.84215	.00105
.2160	.15890	.7840	.84110	.00105
.2170	.15995	.7830	.84005	.00105
.2180	.16100	.7820	.83900	.00106
.2190	.16206	.7810	.83794	.00106
.2200	.16312	.7800	.83688	.00106
.2210	.16418	.7790	.83582	.00106
.2220	.16524	.7780	.83476	.00105
.2230	.16629	.7770	.83371	.00106
.2240	.16735	.7760	.83265	.00106
.2250	.16841	.7750	.83159	.00107

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.2260	.16948	.7740	.83052	
.2270	.17055	.7730	.82945	.00107
.2280	.17162	.7720	.82838	.00107
.2290	.17268	.7710	.82732	.00106
.2300	.17375	.7700	.82625	.00107
.2310	.17482	.7690	.82518	.00107
.2320	.17590	.7680	.82410	.00108
.2330	.17698	.7670	.82302	.00108
.2340	.17805	.7660	.82195	.00107
.2350	.17913	.7650	.82087	.00108
.2360	.18021	.7640	.81979	.00108
.2370	.18129	.7630	.81871	.00109
.2380	.18238	.7620	.81762	.00108
.2390	.18346	.7610	.81654	.00109
.2400	.18455	.7600	.81545	.00109
.2410	.18564	.7590	.81436	.00109
.2420	.18673	.7580	.81327	.00109
.2430	.18782	.7570	.81218	.00109
.2440	.18891	.7560	.81109	.00110
.2450	.19001	.7550	.80999	.00109
.2460	.19110	.7540	.80890	.00110
.2470	.19220	.7530	.80780	.00110
.2480	.19330	.7520	.80670	.00110
.2490	.19440	.7510	.80560	.00110
.2500	.19550	.7500	.80450	.00110

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.2510	.19660	.7490	.80300	.00111
.2520	.19771	.7480	.80229	.00111
.2530	.19882	.7470	.80118	.00110
.2540	.19992	.7460	.80008	.00111
.2550	.20103	.7450	.79897	.00111
.2560	.20214	.7440	.79786	.00111
.2570	.20325	.7430	.79675	.00112
.2580	.20437	.7420	.79563	.00111
.2590	.20548	.7410	.79452	.00112
.2600	.20660	.7400	.79340	.00112
.2610	.20772	.7390	.79228	.00112
.2620	.20884	.7380	.79116	.00112
.2630	.20996	.7370	.79004	.00112
.2640	.21108	.7360	.78892	.00112
.2650	.21220	.7350	.78780	.00113
.2660	.21333	.7340	.78667	.00113
.2670	.21446	.7330	.78554	.00112
.2680	.21558	.7320	.78442	.00113
.2690	.21671	.7310	.78329	.00113
.2700	.21784	.7300	.78216	.00113
.2710	.21897	.7290	.78103	.00113
.2720	.22010	.7280	.77990	.00114
.2730	.22124	.7270	.77876	.00113
.2740	.22237	.7260	.77763	.00114
.2750	.22351	.7250	.77649	.00113

Arrow	*Area of Segmēt.*	*Arrow*	*Area of Segmēt.*	*Common Differences*
.2760	.22464	.7240	.77536	.00114
.2770	.22578	.7230	.77422	.00114
.2780	.22692	.7220	.77308	.00114
.2790	.22806	.7210	.77194	.00114
.2800	.22920	.7200	.77080	.00115
.2810	.23035	.7190	.76965	.00115
.2820	.23150	.7180	.76850	.00114
.2830	.23264	.7170	.76736	.00115
.2840	.23379	.7160	.76621	.00115
.2850	.23494	.7150	.76506	.00115
.2860	.23609	.7140	.76391	.00115
.2870	.23724	.7130	.76276	.00115
.2880	.23839	.7120	.76161	.00116
.2890	.23955	.7110	.76045	.00115
.2900	.24070	.7100	.75930	.00116
.2910	.24186	.7090	.75814	.00116
.2920	.24302	.7080	.75698	.00116
.2930	.24418	.7070	.75582	.00116
.2940	.24534	.7060	.75466	.00116
.2950	.24650	.7050	.75350	.00116
.2960	.24766	.7040	.75234	.00116
.2970	.24882	.7030	.75118	.00116
.2980	.24998	.7020	.75002	.00117
.2990	.25115	.7010	.74885	.00117
.3000	.25232	.7000	.74768	.00116

Arrow	Area of Segmẽt.	Arrow	Area of Segmẽt.	Common Differences.
.3010	.25348	.6990	.74652	.00117
.3020	.25465	.6980	.74535	.00117
.3030	.25582	.6970	.74418	.00117
.3040	.25699	.6960	.74301	.00117
.3050	.25816	.6950	.74184	.00117
.3060	.25933	.6940	.74067	.00118
.3070	.26051	.6930	.73949	.00118
.3080	.26169	.6920	.73831	.00117
.3090	.26286	.6910	.73714	.00118
.3100	.26404	.6900	.73596	.00118
.3110	.26522	.6890	.73478	.00118
.3120	.26640	.6880	.73360	.00118
.3130	.26758	.6870	.73242	.00118
.3140	.26876	.6860	.73124	.00118
.3150	.26994	.6850	.73006	.00118
.3160	.27112	.6840	.72888	.00119
.3170	.27231	.6830	.72769	.00118
.3180	.27349	.6820	.72651	.00119
.3190	.27468	.6810	.72532	.00119
.3200	.27587	.6800	.72413	.00119
.3210	.27706	.6790	.72294	.00119
.3220	.27825	.6780	.72175	.00119
.3230	.27944	.6770	.72056	.00119
.3240	.28063	.6760	.71937	.00119
.3250	.28182	.6750	.71818	.00119

Arrow	Area of Segmēt.	Arrow	Area of Segmēt.	Common Differences.
.3260	.28301	.6740	.71699	
.3270	.28421	.6730	.71579	.00120
.3280	.28540	.6720	.71460	.00119
.3290	.28660	.6710	.71340	.00120
.3300	.28780	.6700	.71220	.00120
				.00119
.3310	.28899	.6690	.71101	
.3320	.29019	.6680	.70981	.00120
.3330	.29139	.6670	.70861	.00120
.3340	.29259	.6660	.70741	.00120
.3350	.29379	.6650	.70621	.00120
				.00120
.3360	.29499	.6640	.70501	
.3370	.29619	.6630	.70381	.00120
.3380	.29739	.6620	.70261	.00120
.3390	.29850	.6610	.70140	.00121
.3400	.29981	.6600	.70019	.00121
				.00121
.3410	.30102	.6590	.69898	
.3420	.30223	.6580	.69777	.00121
.3430	.30344	.6570	.69656	.00121
.3440	.30465	.6560	.69535	.00121
.3450	.30586	.6550	.69414	.00121
				.00121
.3460	.30707	.6540	.69293	
.3470	.30828	.6530	.69172	.00121
.3480	.30949	.6520	.69051	.00121
.3490	.31070	.6510	.68930	.00121
.3500	.31191	.6500	.68809	.00121

Arrow	Area of Segmẽt.	Arrow	Area of Segmẽt.	Common Differences.
.3510	.31313	.6490	.68687	
.3520	.31435	.6480	.68565	.00122
.3530	.31557	.6470	.68443	.00122
.3540	.31678	.6460	.68322	.00121
.3550	.31800	.6450	.68200	.00122
				.00122
.3560	.31922	.6440	.68078	.00122
.3570	.32044	.6430	.67956	.00122
.3580	.32166	.6420	.67834	.00122
.3590	.32288	.6410	.67712	.00122
.3600	.32410	.6400	.67590	.00123
.3610	.32533	.6390	.67467	.00122
.3620	.32655	.6380	.67345	.00122
.3630	.32777	.6370	.67223	.00123
.3640	.32900	.6360	.67100	.00123
.3650	.33023	.6350	.66977	.00123
.3660	.33146	.6340	.66854	.00123
.3670	.33269	.6330	.66731	.00123
.3680	.33392	.6320	.66608	.00123
.3690	.33515	.6310	.66485	.00122
.3700	.33637	.6300	.66363	.00122
.3710	.33759	.6290	.66241	.00123
.3720	.33882	.6280	.66118	.00123
.3730	.34005	.6270	.65995	.00123
.3740	.34128	.6260	.65872	.00123
.3750	.34251	.6250	.65749	.00124

Arrow	Area of Segmẽt.	Arrow	Area of Segmẽt.	Common Differences.
.3760	.34375	.6240	.65625	
.3770	.34499	.6230	.65501	.00124
.3780	.34623	.6220	.65377	.00124
.3790	.34746	.6210	.65254	.00123
.3800	.34869	.6200	.65131	.00123
				.00124
.3810	.34993	.6190	.65007	
.3820	.35117	.6180	.64883	.00124
.3830	.35240	.6170	.64760	.00123
.3840	.35364	.6160	.64636	.00124
.3850	.35488	.6150	.64512	.00124
				.00124
.3860	.35612	.6140	.64388	
.3870	.35736	.6130	.64264	.00124
.3880	.35860	.6120	.64140	.00124
.3890	.35984	.6110	.64016	.00124
.3900	.36108	.6100	.63892	.00124
				.00124
.3910	.36232	.6090	.63768	
.3920	.36357	.6080	.63643	.00125
.3930	.36481	.6070	.63519	.00124
.3940	.36606	.6060	.63394	.00125
.3950	.36731	.6050	.63269	.00125
				.00124
.3960	.36855	.6040	.63145	
.3970	.36979	.6030	.63021	.00124
.3980	.37104	.6020	.62896	.00125
.3990	.37228	.6010	.62772	.00124
.4000	.37353	.6000	.62647	.00125
				.00125

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.4010	.37478	.5990	.62522	
.4020	.37603	.5980	.62397	.00125
.4030	.37728	.5970	.62272	.00125
.4040	.37853	.5960	.62147	.00125
.4050	.37978	.5950	.62022	.00125
.4060	.38103	.5940	.61897	.00125
.4070	.38228	.5930	.61772	.00125
.4080	.38353	.5920	.61647	.00125
.4090	.38478	.5910	.61522	.00125
.4100	.38603	.5900	.61397	.00125
.4110	.38728	.5890	.61272	.00126
.4120	.38854	.5880	.61146	.00125
.4130	.38979	.5870	.61021	.00125
.4140	.39104	.5860	.60896	.00126
.4150	.39230	.5850	.60770	.00125
.4160	.39355	.5840	.60645	.00126
.4170	.39481	.5830	.60519	.00125
.4180	.39606	.5820	.60394	.00126
.4190	.39732	.5810	.60268	.00126
.4200	.39858	.5800	.60142	.00126
.4210	.39984	.5790	.60016	.00125
.4220	.40109	.5780	.59891	.00126
.4230	.40235	.5770	.59765	.00126
.4240	.40361	.5760	.59639	.00126
.4250	.40487	.5750	.59513	.00126

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
.4260	.40613	.5740	.59387	.00126
.4270	.40739	.5730	.59261	.00126
.4280	.40865	.5720	.59135	.00126
.4290	.40991	.5710	.59009	.00126
.4300	.41117	.5700	.58883	.00126
.4310	.41243	.5690	.58757	.00126
.4320	.41369	.5680	.58631	.00126
.4330	.41495	.5670	.58505	.00126
.4340	.41621	.5660	.58379	.00126
.4350	.41747	.5650	.58253	.00127
.4360	.41874	.5640	.58126	.00126
.4370	.42000	.5630	.58000	.00126
.4380	.42126	.5620	.57874	.00126
.4390	.42252	.5610	.57748	.00126
.4400	.42378	.5600	.57622	.00127
.4410	.42505	.5590	.57495	.00126
.4420	.42631	.5580	.57369	.00127
.4430	.42758	.5570	.57242	.00127
.4440	.42885	.5560	.57115	.00126
.4450	.43011	.5550	.56989	.00127
.4460	.43138	.5540	.56862	.00127
.4470	.43265	.5530	.56735	.00126
.4480	.43391	.5520	.56609	.00127
.4490	.43518	.5510	.56482	.00126
.4500	.43644	.5500	.56356	.00127

Arrow	*Area of Segmẽt.*	*Arrow*	*Area of Segmẽt.*	*Common Differences.*
·4510	·43771	·5490	·56229	
·4520	·43898	·5480	·56102	·00127
·4530	·44024	·5470	·55976	·00126
·4540	·44150	·5460	·55850	·00126
·4550	·44277	·5450	·55723	·00127
				·00127
·4560	·44404	·5440	·55596	
·4570	·44531	·5430	·55469	·00127
·4580	·44658	·5420	·55342	·00127
·4590	·44785	·5410	·55215	·00127
·4600	·44912	·5400	·55088	·00127
				·00127
·4610	·45039	·5390	·54961	
·4620	·45166	·5380	·54834	·00127
·4630	·45293	·5370	·54707	·00127
·4640	·45420	·5360	·54580	·00127
·4650	·45547	·5350	·54453	·00127
				·00127
·4660	·45674	·5340	·54326	
·4670	·45801	·5330	·54199	·00127
·4680	·45928	·5320	·54072	·00127
·4690	·46055	·5310	·53945	·00127
·4700	·46183	·5300	·53817	·00128
				·00127
·4710	·46310	·5290	·53690	
·4720	·46437	·5280	·53563	·00127
·4730	·46564	·5270	·53436	·00127
·4740	·46691	·5260	·53309	·00127
·4750	·46818	·5250	·53182	·00127
				·00128

Arrow	Area of Segmẽt.	Arrow	Area of Segmẽt.	Common Differences.
.4760	.46946	.5240	.53054	.00127
.4770	.47073	.5230	.52927	.00127
.4780	.47200	.5220	.52800	.00127
.4790	.47327	.5210	.52673	.00127
.4800	.47454	.5200	.52546	.00127
.4810	.47581	.5190	.52419	.00127
.4820	.47708	.5180	.52292	.00127
.4830	.47835	.5170	.52165	.00128
.4840	.47963	.5160	.52037	.00127
.4850	.48090	.5150	.51910	.00128
.4860	.48218	.5140	.51782	.00127
.4870	.48345	.5130	.51655	.00127
.4880	.48472	.5120	.51528	.00128
.4890	.48600	.5110	.51400	.00127
.4900	.48727	.5100	.51273	.00127
.4910	.48854	.5090	.51146	.00127
.4920	.48981	.5080	.51019	.00128
.4930	.49109	.5070	.50891	.00127
.4940	.49236	.5060	.50764	.00128
.4950	.49364	.5050	.50636	.00127
.4960	.49491	.5040	.50509	.00127
.4970	.49618	.5030	.50382	.00127
.4980	.49745	.5020	.50255	.00128
.4990	.49873	.5010	.50127	.00127
.5000	.50000	.5000	.50000	

The Conſtruction of this Table is laid down in *Probl.* 4. *Pag.* 5. and part of its utility appears in the Solution of the Queſtion at the bottom of *Pag.* 7. as alſo in the Solutions of *Probl.* 34. and 63.

The Uſe of this Table primarily conduces to the untying of that Knot which (on all hands) is acknowledged the moſt difficult in the whole Myſtery of *Gauging*, *viz.* to make (by Multiplication only) a practical and very nigh approximation to the Gauging of the vacant or replete Fruſtums of Wine, Brandy, Ale, or Oil Cask (partly empty) the Cask being (as moſt generally they are) aſſumed for the Middle Fruſtum of a Spheroid, intercepted between two Planes parallel, cutting the Axe at Right Angles, the Liquor cutting the Heads, and the Casks Axe poſited parallel to the Horizon.

The Uſe and Utility of this Table will appear in the moſt facile and practicable Solutions (*inter alia*) of theſe Six uſeful and abſtruſe Problems, the moſt intricate in the whole Art Metrical. *Viz*,

I. Probl.

Having a Casks liquid Capacity, and the Boung Diameter, together with its dry, or wet part; To find the Vacuity, &c.

II. Probl.

Having a Casks liquid Capacity, and the Boung Diameter, together with the Vacuity, or the quantity of Liquor remaining in the Cask; To find the Boung Diameters dry, or wet part.

III. Probl.

Having the liquid Capacity of a Cylindrical Cask, together with the Diameter of its Base, and its wet or dry part; To find the Vacuity, &c. and the Converse.

IV. Probl.

Having the Diameter, or Area of a Circle; To find the Area of that Circles Segment, correspondent to any part of the Diameter inquired of.

V. Probl.

Having a Casks Boung Diameter, and liquid Capacity, together with the Vacuity, &c. at any assigned Altitude of the remaining Liquor (as taken for the middle Frustum of a Spheroid) To find either of the Casks correspondent Frustums, if taken, as the middle Frustum of a Parabolical Spindle.

VI.

VI. PROBL.

Having a Casks Dimenſions, and liquid Capacity, together with the Vacuity, &c. at any altitude of Liquor given, as taken for the middle Fruſtum of a Spheroid; To find either of the correſpondent Fruſtums of the Cask, if taken for the middle Fruſtum of two Cones, abutting upon one common Baſe, intercepted between two Planes, parallel, and cutting the Axe at Right Angles.

Of the CONSTRUCTION, &c. *of the* CYCLOMETRICAL TABLES, *in Pages* 35 *and* 50.

To examine or try any Number in the Tables, or to Calculate any other, (not therein compriſed) by a ſingle Multiplication; if the Circles Area be ſought in Wine Gallons, the Square of the Diameter, multiplied by .0033999204, but (for contraction) by .0034-|-, (being the natural number of the Arithmetical Complement of the Logarithm of 294.1116, the Product of 1.2732,39549, the Square of the Diameter of that Circle, whoſe Area is unity, multiplied by 231, (the number of Cubical Inches in a Wine Gallon) produces the Circles Area in Wine Gallons, being the Tabular number

ber ſought. But when the Circles Area is deſired in Ale Gallons; then, multiply the Square of the Diameter by .002785099866, or rather by .0027851+, and the Product (ſo accruing) is the Circles Area in Ale Gallons, required: Where note, That this common complicated Multiplicator .0027851+ is the natural number of the Complement Arithmetical of the Logarithm of 359.0538+ the Product of 1.2732,39549 the Square of the Diameter of that Circle, whoſe Area is Unity, multiplied by 282, the Cubical Inches in an Ale or Beer Gallon.

Note here, That in the Table for Wine Meaſure the Area of any Circle (under 61 Inches Diameter,) and for Ale Meaſure (under 217 Inches,) is in effect given, unto the centeſſimal part of an Inch of the Diameter (provided the laſt Figure in the Decimal Fraction be 5.)

As for INSTANCE.

To find that Circles Area in Wine Meaſure, whoſe Diameter is 17.15 *Inches.*

Area of that Circ. whoſe Diam. is 17.2 *per* Tab.	1.0059	diff. .0117
And of that, whoſe Diam. is 17.1 *per* Tab.	0.9942	
Then to that of 17.1 I add the Semidifference	0.0058	
So Area of Circle whoſe Diam. is 17.15 appears	1.0000	Wine Gall. exactly.

Whereby

Whereby it appears, that a Cylindrical Veſſel of 17.15 Inches Diameter, doth upon every Inch of its Depth, or Altitude, contain one entire Wine Gallon, or 231 Cubical Inches; and of 171.5 Inches Diameter, 100 Wine Gallons; and of 1715 Inches Diameter, 1000 Wine Gallons, upon every Inch.

Some Uſes and Emprovements of the CYCLOMETRICAL TABLE.

Having the Dimenſions of a Cask, aſſum'd for the middle Fruſtum, or Zone of a Spheroid; To Gauge its whole Capacity.

As for INSTANCE.

A Casks Boung Diameter D, m, = 21.5 *Inches, Head Diameter* w, d, *or* u, d, = 18.3, *and* i, o, *the Length* = 39 *Inches; To find its whole Capacity in Wine Gallons.* *Fig.*12,13.

	Wine Gall.
Of *D, m*, 21.5 *per* Tab. doubled Area =	3.1432
Of *w, d*, 18.3 *per* Table Area =	1.1386
Tripled Area of the Mean Circle =	4.2818
$\frac{1}{3}$ of *i, o*, the Length 39 =	13
The Casks whole Capacity =	55.6634

being 55 $\frac{1}{2}$ Gall. and 1 $\frac{1}{3}$ Pint, *proximè*.

If

If the Cask ſtand on the Head, with the Diameters parallel to the Horizon, the ſame (as alſo a Spheroidal Tunn) may by the Table be alſo Inched (if need require) by Rule in *Pag.* 137 and 133.

Having the Dimenſions of a Brewers Tunn, or Veſſel,

1. *Conical,*
2. *Cylindrical,*
3. *Spheroidal,*
4. *Elliptical, conſiſting either of equal, or unequal Baſes;*
5. *Elliptical at one Baſe, and Circular at the other:*

To find the whole liquid Capacity after a more expeditious, facile, and practical way, than any other yet extant; and alſo to Inch the ſame.

Fig. 9. 1. *A Conical Tunns upper Diameter is* 215 *Inches, the other* = 209.6, *and the Depth* 50 *Inches: To find its Capacity, in Ale Gallons, by the Table.*

Leſſer Diameter = 209.6
Semidiff. of Diam. = 2.7

	Ale Gall.
Diam. in mid of dep. = 212.3 *per* T.	125.5280
$\frac{1}{3}$ of Area of Semidiff. 2.7 *per* Tab. =	.0068
Reduced into a Cylinder, whoſe Area	125.5348
Depth =	50
The Tunns whole Capacity =	6276.7400

being 174 Beer Bar. 12 Gall. 3 Quarts, *prox.*

Another INSTANCE.

A Conical Tunns greater Diameter, a, b, = 190 *Inches*; *leſſer Diameter*, c, d, = 140; *and* n, o, *the Depth*, = 60 *Inches*: *To find this Tunns Capacity, in Ale Gallons.* Fig. 9.

Leſſer Diameter, *c*,*d*, = 140
Semidiff. of *a*,*b*, & *c*,*d*, = 25

	Ale Gall.
Diam. in mid. of dep. = 165 *per* T. =	75.8243
$\frac{1}{3}$ of Area of Semidiff. 25 is *per* T. =	.5803
Reduc'd into a Cylinder, whoſe Area	76.4046
Depth =	60
The Tunns whole Capacity =	4584.2760

being 127 Beer Bar. 12 Gall. 1 Quart, *&c.*

For finding the Drip or Fall of theſe, or any other Tunns whatſoever; and for Inching Conical Tunns by the Table; *vide Probl.* 40, 41.

2. *Having the Diameter of the Baſe of a Cylindrical Tunn* = 171.5, *and Depth* = 60 *Inches*; *To find the Tunns whole Capacity, in Ale Gallons, and Inch by Inch.* Fig. 8.

	Ale Gall.
Area of that Circle, whoſe Diam. is 171.5 *per* Tab.	81.9161 *per* Inch.
The Depth =	60
Tunns whole Capacity =	4914.9660

being 136 $\frac{1}{2}$ Beer Bar. 3 Quarts, 1 $\frac{3}{4}$ Pint, *ferè*.

3. *Having*

Fig. 12. 3. *Having the Dimenſions of a Spheroidal Tunn, or Veſſel, as* D, C, m, w, i, d: *To find the Tunns liquid Capacity.*

As for INSTANCE.

Let D, C, m, *the greater Diameter, be* = 144 *Inches*; d, i, w, *the leſſer Diameter* = 108; *and* C, i, *the Depth, or Altitude,* = 60 *Inches.*

	Ale Gall.
Of *D, C, m, per* T. Area, doubled, =	115.5036
And of *d, i, w, per* Tab. Area =	32.4854
Tripled Area of the Mean Circle	147.9890
$\frac{1}{3}$ of *C, i*, the Depth, =	20
The Tunns whole Capacity =	2959.7800

being 82 Beer Bar. 7 Gall. 3 Quarts, and $\frac{1}{4}$ of a Pint, *proximè*.

This, and ſuch like Veſſels, may be Inched alſo by the Table, as *per* Rule and Example in *Pag.* 133. and 137.

Fig. 15. 4. *Having the Dimenſions of* a, b, d, c, (*a Tunn in the form of an Elliptical Solid, conſiſting of two parallel and equal Baſes, alike ſituate*) *the Axe above*, viz. a, b, *being poſited directly oppoſite to* c, d, *the Axe below*; *To find its whole liquid Capacity.*

Let

Let the Axe, a, b, *be* = c, d, *and each* 130 *Inches*; *and the leſſer Diameters*, m, p, *or* u, x, = 110; *and* n, o, *the Depth*, = 60 *Inches.*

a,b,=130. m,p,=110 } *Rectangular Conjugates* { *above.* *below.*
c,d,=130. u,x,=110 }

The leſſer Diam. *m*, *p*, = 110
Semidiff. of Diameters = 10

	Ale Gall.
Tunns leſſ. Diam in midd. 120 *per* T.	40.1054
Area of the Semidiff. 10 *per* T. =	.2785
Area of a Mean Circle =	39.8269
The Depth =	60
The Tunns whole Capacity =	2389.6140

being 66 Beer Barrels, 13 ½ Gallons, and 1 Pint, *proximè.*

Note here, That in caſe the Conjugates of the reſpective Baſes had not been in the Queſtion propoſed, Rectangular and alike ſituate, or if the Poſition of the Conjugates had not been given; the Queſtion then, had been unlimited; and capable of many hundred Anſwers; whereof that (above-given) had been the leaſt, and 66 Beer Barrels, 24 Gallons, 3 Quarts, the Tunns greateſt Capacity; *viz.* when the ſituation of the Conjugates is inverted, that is, when *a*, *b*, the Axe above, is poſited directly oppoſite to *n*, *x*, the leſſer Diameter, below,

below, *&c.* which said greatest Capacity may be thus found, *viz.*

m,p, = 110. a,b, = 130 } *Conjugates* { *above.*
c, d, = 130. u,x, = 110 } { *below.*

$$
\begin{array}{rr}
m, p, = & 110 \\
\frac{1}{2}\ c, d, = & 65 \\
\hline
 & 175 \\
a, b, = & 130 \\
\hline
\text{The first Product} = & 22750
\end{array}
$$

$$
\begin{array}{rr}
c, d, = & 130 \\
\frac{1}{2}\ m, p, = & 55 \\
\hline
 & 185 \\
x, u, = & 110 \\
\hline
\text{The second Product} = & 20350 \\
\text{The first Product} = & 22750 \\
\hline
\text{Their Summ} = & 43100
\end{array}
$$

Multiplicator for Ale Gall. = .0009283666 +

$$
\begin{array}{r}
43100 \\
\hline
928366600 \\
27850998 \\
37134664 \\
\hline
40.0126004600 \\
\text{The Depth} = 60 \\
\hline
\text{Tunns gr. Capac. in Ale Gall.} 2400.7560276
\end{array}
$$

An

An Elliptical Tunn, having inequality of Baſes at the top and bottom, is either the Fruſtum of an Elliptical Cone, or Pyramid, or a Cylindroid; the one is, when the Rectangular Conjugates, above and below, are reciprocally proportional; and the other, when they are diſproportional.

Having the Dimenſions of an Elliptical Tunn, adjudg'd the Fruſtum of an Elliptical Cone, or Pyramid; the Rectangular Conjugates above, viz. a, b, *and* m, p; *and below, to wit,* c, d, *and* x, u; *being proportional, parallel, and alike ſituate;* n, o, *the Depth,* = 20 *Inches.* Fig. 15.

This Veſſel (or any of like nature) may be Gauged by the Rule in *Pag.* 148. and thereby the Tunns Capacity found = 1164.543 Ale Gall. the Dimenſions being known, *viz.*

a,b,=200. m,p,=160 } *Conjugates* { *above.*
c,d,=120. x, u,= 96 } { *below.*

	c, *d*, =	120
Semidifference of *c*, *d*, and *a*, *b*, =		40
The Axe in the midſt of the depth =		160
	x, *u*, =	96
Semidifference of *x*, *u*, and *m*, *p*, =		32
Leſſer Diam. in midſt of Tunns depth =		128
Semidiff. of Diam. in midſt of the Tunn =		16
	Summ =	144

	Ale Gall.
144 *per* Tab. =	57.7518
Area of Semidiff. 16 *per* Tab. =	.7130
Remainder =	57.0388
Semidiff. of *a, b,* and *m, p,* 20 *per* T. =	1.1140
Area of $\frac{1}{4}$ of Semidiff. *viz.* 5 *per* T. =	.0696
Summ =	58.2224
Depth =	20
The Tunns whole Capacity =	1164.4480

Fig. 15. *Having the Dimensions of a Tunn in the form of an Elliptical Solid, (called a Cylindroid) as* a, b, d, c, *the Rectangular Conjugates above and below, being unequal, and disproportional; but the Bases, parallel, and alike situate, the Axe above being posited directly opposite to the Axe below; and the Depth* n, o, = 20 *Inches.*

a,b,=200. m,p,=160 } *Rectangular Conjugates* { *above.*
c,d,=120. x,u,=100 } *below.*

The whole Capacity of this Tunn is (by Rule in *Pag.* 148.) found = 1180.8824 Ale Gallons, being 32 $\frac{5}{4}$ Beer Bar. 1 Gallon, 3 Quarts, and 1 Pint, *proximè.*

To Gauge such Tunns by the Table, the Rule is:

1. Unto *c, d,* (the Axe of the lesser Base,) add the Semidifference between that and *a, b,* the

the Axe of the greater Baſe; (which done) that Summ is equal to the Axe of the Arithmetical Mean Baſe, or the Veſſels Axe in the midſt of its depth; which reſerve.

2. Unto *x*, *u*, (the leſſer Diameter of the leaſt Baſe) add the Semidifference between that and *m*, *p*, (the leſſer Diameter of the greater Baſe) and that Summ is equal to the Veſſels leſſer Diameter in the midſt of its depth.

3. Unto this leſſer Diameter, in the midſt of the Tunn, add the Semidifference between that and the Axe in the midſt of the Tunn reſerved; then unto that Summ (as a Diameter) inſert from the Table its proper Area; and under it, the Area proper to the Semidifference (laſt mentioned) ſubducting the leſſer from the greater, and noting the Remainder.

Laſtly, Unto that Remainder add the Area (to be found in the Table) proper to the Semidifference of the Veſſels greateſt Diameters, to wit, *a*, *b*, and *m*, *p*; (which done) that Summ is equal to the Area of a Mean Circular Baſe, equal in Capacity unto that of the Elliptical Baſe, in the midſt of the Tunns depth, reducing the Veſſel into a Cylinder (*proximè verum*,) which multiplied by the Depth produces the Tunns whole Capacity, in Ale Gallons, the thing required.

c, *d*, = 120

Semidifference of *c*, *d*, and *a*, *b*, = 40

The Axe in the middle = 160

$x, u, = 100$
Semidifference $x, u,$ and $m, p, = 30$

Leſſer Diameter in the midſt $= 130$
Semidiff. of Diam. in the middle of Tunn $= 15$

Summ $= 145$

	Ale Gall.
145 *per* Tab. $=$	58.5567
Semidiff. of Diam. in middle *per* T. $=$	.6266
Remainder $=$	57.9301
Semidiff. of *a, b,* and *m, p,* 20 *per* T. $=$	1.1140
Area of a Mean Circular Baſe $=$	59.0441
The Depth $=$	20
The Veſſels whole Capacity $=$	1180.8820

But in caſe the Conjugates of the reſpective Baſes had not been expreſs'd in the Queſtion, Rectangular and alike ſituate, the Queſtion had not been limitted, and ſo capable of many Anſwers, whereof that (given already) is one, and the leaſt; the Tunns greateſt Capacity, to wit, when the ſituation or poſition of the Conjugates is inverted, *a, b,* the Axe above, being placed againſt *x, u,* the leſſer Diameter below, is thus found, *viz.*

m, p,

m,p, = 160. a,b, = 200 } *Rectangular* } *above.*
c,d, = 120. x,u, = 100 } *Conjugates* } *below.*

m, *p*, =	160
½ *c*, *d*, =	60
Summ =	220
a, *b*, =	200
The first Product =	44000

c, *d*, =	120
½ *m*, *p*, =	80
Summ =	200
x, *u*, =	100
The second Product =	20000
The first Product =	44000
Their Summ =	64000

Multiplicator for Ale Gall. .000928366+
64000

37134664
55701996

59.4154624000
The Depth = 20

Tunns greateſt Capac. = 1188,3092480
Ale Gallons.

5. *Having the Dimensions of a Tunn, Elliptical above, and Circular below; as* a, b, d, c.

Fig. 15. *Let* a, b, *the Axe of the Elliptical Base, be* = 210; *and* m, p, *the lesser Diameter* = 140; *and* c, d, *or* u, x, *the Diameter of the Circular Base, also* = 210; *and* n, o, *the Depth,* = 24 *Inches: To find the Tunns liquid Capacity by the Cyclometrical Table.*

Vide Pag. 156.

Lesser Diam. of Elliptical Base *m*, *p*, = 140
Semidifference of *a*, *b*, and *m*, *p*, = 35

Lesser Diameter in middle of the Tunn = 175

	Ale Gall.
175 *per* Tab. =	85.2937
Semidiff. of *a*, *b*, and *m*, *p*, or whole Differ. of Diam. in middle of the Tunn, 35 *per* T.	3.4117
Differ. of *a*, *b*, and *m*, *p*, 70 *per* T. =	13.6470
Area of a Circular Base = Tunns mixt Base in midst of the depth	102.3524
n, *o*, the Depth =	24
The Tunns whole Capacity =	2456.4576

Avertisement.

It is very necessary in all Tunns (and in these more especially) that the Diameters, and cross Diameters, be Instrumentally taken with much pre-

preciſeneſs, unto every three Inches (at the leaſt,) but rather unto every Inch of the Depth; by which it will appear, how far the Veſſel is Circular, and how far Elliptical, *&c.* otherwiſe, by reaſon of warping of the Tunns ſides, and other irregularities, exceeding great Errours may ſurprize the Gauger, though otherwiſe very expert.

There is a Tunn, as c, a, d, b, p, o, n, *whereof the Baſes are Rectangular Parallelograms, unequal, but parallel and alike ſituate, and the ſides, diſproportional, (called a* Priſmoid) c, b, *the length above, being poſited directly oppoſite to* n, o, *the length below,* &c. *the depth* b, o, = 30 *Inches: How many Beer Barrels are this Tunns whole Capacity; and how many Beer Barrels remain in the Tunn, when* 5 *Inches are dry?* Fig. 4.

[Gauge the whole Tunn by the Rule in *Pag.* 148. and Inch the ſame by the Rule *Pag.* 149.]

The Dimenſions are theſe, viz.

c, b, = 260. b, d, = 140 } *Rectangular* } *above.*
n, o, = 210. o, p, = 110 } *Conjugates* } *below.*

$$\begin{array}{r} c,\ b, = 260 \\ \frac{1}{2}\ n,\ o, = 105 \\ \hline \text{Summ} = 365 \end{array}$$

$$
\begin{array}{rr}
 & 365 \\
b, d, = & 140 \\
\hline
\text{The first Product} = & 51100
\end{array}
$$

$$
\begin{array}{rr}
n, o, = & 210 \\
\frac{1}{2}\ c, b, = & 130 \\
\hline
\text{Summ} = & 340 \\
o, p, = & 110 \\
\hline
\text{The second Product} = & 37400 \\
\text{The first Product} & 51100 \\
\hline
\text{Their Summ} = & 88500
\end{array}
$$

Multiplicator for Beer Bar. ,0000328245 6+

$$
\begin{array}{r}
88500 \\
\hline
2.904973560 00 \\
30 \\
\hline
\end{array}
$$

$b, o,$ the Depth =

Resp. The whole Capacity = 87.14940; and 69 Barrels $\frac{3}{4}$, and 3 Gallons, at 5 dry Inches.

In the first Column of the following Table, is set down the dry Inches; in the second, the Beer Barrels remaining in the Tunn; in the third, the Decimal Parts above a Barrel; and in the fourth or last, the Decimal Parts reduced into Gallons.

0	87	.1559	5 $\frac{3}{4}$
1	83	.5951	21 $\frac{1}{4}$
2	80	.0825	3
3	76	.6179	22
4	73	.2009	7 $\frac{3}{4}$
5	69	.8312	30
6	66	.5086	18
7	63	.2325	8 $\frac{1}{2}$
8	60	.0028	00
9	56	.8191	29 $\frac{1}{2}$
10	53	.6809	24 $\frac{3}{4}$
11	50	.5882	21
12	47	.5405	19 $\frac{3}{4}$
13	44	.5374	19 $\frac{1}{2}$
14	41	.5787	20 $\frac{3}{4}$
15	38	.6640	24
16	35	.7930	28 $\frac{3}{4}$
17	32	.9653	34 $\frac{3}{4}$
18	30	.1806	6 $\frac{3}{4}$
19	27	.4386	15 $\frac{3}{4}$
20	24	.7390	26 $\frac{1}{2}$
21	22	.0814	3
22	19	.4655	16 $\frac{3}{4}$
23	16	.8910	32 $\frac{1}{2}$
24	14	.3575	12 $\frac{3}{4}$
25	11	.8547	31
26	9	.4123	15
27	7	.0000	00
28	4	.6274	22 $\frac{1}{2}$
29	2	.2942	10 $\frac{3}{4}$
30	0	.0000	00

Obſerve here, That if the Conjugates of the reſpective Baſes had not (in the Queſtion laſt propoſed) been expreſſed (Rectangular and alike ſituate) the Queſtion then had been unlimitted, and capable of many Anſwers, whereof 87 Barrels and $5\frac{3}{4}$ Gallons is one and the leaſt, and 93 Barrels and 2 Gallons, the greateſt Anſwer; to wit, this Tunns Capacity, by inverſion of the ſituation or poſition of the Conjugates, that is, when *c*, *b*, the length above, is poſited directly oppoſite unto *o*, *p*, the breadth below, *&c.*

The greateſt Capacity is thus found, viz.

b, d, = 140. c, b, = 260
n, o, = 210. o, p, = 110

b, *d*, = 140
$\frac{1}{2}$ *n*, *o*, = 105

Summ = 245
c, *b*, 260

14700
490

The firſt Product = 63700

n, *o*, = 210
$\frac{1}{2}$ *b*, *d*, = 70

Summ = 280

o, *p*,

280
o, p, = 110

The ſecond Product = 30800
The firſt Product = 63700

Their Summ = 94500

Multiplicator for Beer Bar. .000032824564
94500

1641228000
13129824
29542104

3.101920920000
The Depth = 30

Tunns greateſt Capacity = 93.057630; being 93 Beer Barrels, 2 Gallons, and $\frac{3}{40}$ of a Gallon, *proximè*.

Yet I ſuppoſe this Notion may be oftner uſeful to the Timber Merchant, and Carpenter, than to the Gauger.

There is a Tunn, as c, a, d, b, p, o, n, *conſiſting of heterogeneal Baſes, (alſo called a* Priſmoid,) *the Baſe above, being a Rectangular Parallelogram; and that below, a perfect Square; the Conjugates in each Baſe cutting one another at Right Angles; and thoſe above, poſited directly oppoſite to theſe, below: How many Ale Gallons* Fig. 4.

Gallons may the whole Tunn contain, and how many remain in the ſame, when only ſix Inches are dry?

The Dimenſions, viz.

	length.	*breadth.*	
c, a,	= 210	a, d, = 140	*Rectangular* } *above.*
n, o,	= 210	o, p, = 210	*Conjugates* } *below.*

And b, o, *the whole Depth* = 24 *Inches.*

Note, That the Multiplicators (given in *Pag.* 164. for a Pyramidal Square Tunn) are here to be uſed.

The Operation for the whole Capacity.

$$\begin{array}{rl} & n, o, = 210 \\ \frac{1}{2} & a, d, = 70 \\ \hline \text{Summ} = & 280 \\ & n, o, = 210 \\ \hline & 2800 \\ & 560 \\ \hline \text{The firſt Product} = & 58800 \end{array}$$

$$\begin{array}{rl} & a, d, = 140 \\ \frac{1}{2} & n, o, = 105 \\ \hline \text{Summ} = & 245 \end{array}$$

c, a,

245
c, *a*, = 210

2450
490

The second Product = 51450
The first Product = 58800

Their Summ = 110250

Multiplicator for Ale Gall. = .001182033+

110250

59101650
2364066
11820330
1182033

130.319138250
The Depth = 24

Resp. Tunns whole Capacity 3127.6593 *&c.* Ale Gallons.

n, *o*, = 210

a, *d*, = 140

Their Difference = 70 (2.916667+ the common Addend unto *a*, *d*, the breadth, upon every Inch downwards, *viz.*
Whole depth *b*, *o*, = 24

2.916667+

2.916667+

The propoſed dry Inches = 6

17.5

a, d, = 140

The breadth, *a, d,* = 157.5 at 6 Inches diſtance from the top.

So the Tunns Fruſtums Dimenſions are,

c, a, = 210. a, d, = 157.5
n, o, = 210. o, p, = 210

And the wet Inches = 18.

n, o, = 210
½ *a, d,* = 78.75

Summ = 288.75
n, o, = 210

288750
57750

The firſt Product = 60637.5

a, d, = 157.5
½ *n, o,* = 105

Summ = 262.5

c, a,

262.5
$c, a, = 210$

26250
5250

The ſecond Preduct = 55125
The firſt Product = 60637.5

Their Summ = 115762.5
Multiplicator for Ale Gall. = .001182033+

3472875
3472875
23152500
9261000
1157625
1157625

136.835095162 5
The wet Inches = 18

Reſponſ. 2463.03171 *&c.* Ale Gallons in the Tunn.

This, and all ſuch like Veſſels, may be alſo Inched by the Rule for a Circulo-Elliptical Veſſel, in *Pag.* 158.

Of

Of the first Ten Regular POLYGONIAL or MULT-ANGL'D FIGURES.

A Universal Rule for Gauging a whole Pyramidal Tunn, in the form of any of these Ten subsequent Polygonial Solids; and also a Frustum, or part of any such Tunn, at any Depth assign'd, viz.

1. The Difference of the side of the greater Base, from that of the lesser, divided by the Tunns Depth, (or multiplied by its proper Multiplicator) quotes, or produces the lesser sides Increment, or the greaters Decrement, (according to the tenour of the Question) at one Inches distance on the depth, either from the top to the bottom, or contrariwise; which call the common Addend or Subducend: Yet note here, That this is useful only in finding the Capacity of the Tunns Frustum, and of no use in finding the whole Tunns Capacity.

2. From the Square of the Summ of the side at top, and of that at bottom (if the whole Tunns Capacity be sought,) or of the sides of any other two Bases assign'd, (or found by the Rule above given) subduct the Rectangle of those sides; and that Remainder multiply either by the whole Depth, (or by any proposed part of it,) and reserve the Product.

Then

Then

1. If the Tunn propoſed be Pyramidal, and in the form of an Equilateral Triangular Solid; the afore reſerved Product, divided by 6.928203 (the Square Root of 48) or multiplied by .1443376, either quotes, or produces the Cubical Inches equal to the ſolidity of the whole Tunn, or of its Fruſtum (according to the tenour of the Queſtion): but for finding the liquid Capacity, multiply the reſerved Product by

The proper Multiplicator for { Ale Gall. .0005118355+ / Beer Bar .0000142176 5—

2. If the Tunn be a Fruſtum of a Pyramidal Square, then the $\frac{1}{3}$ of the ſaid reſerved Product is equal to the Solidity in Cubical Inches; but for the liquid Capacity, multiply the whole reſerved Product by

The proper Multiplicator for { Ale Gallons.0011820 33+ / BeerBar. .0000328245 6+

3. If the propoſed Veſſel be a Fruſtum of a *Pentagonal Pyramid*; the ſaid reſerved Product divided by 1.7437+, or multiplied by .5734933+, either quotes, or produces the Solidity in Cubical Inches; but for the liquid Capacity,

The proper Multiplicator for { Ale Gallons .0020336 64—. / Beer Bar. .0000564906+

4. If the Veſſel inquired of be a Fruſtum of an *Hexagonal Pyramid*, the ſaid reſerved Product divided by 1.154701+, or multiplied by .8660252+, either quotes, or produces the Solidity in Cubical Inches; but for finding the liquid Capacity,

The proper Multiplicator for { Ale Gall. .003071011— / Beer Bar. .0000853059+

5. If the Tunn be a Fruſtum of an *Heptagonal Pyramid*; the ſaid reſerved Product divided by .8253905 +, or multiplied by 1.211548—, either quotes, or produces the Solidity in Cubical Inches;

The proper Multiplicator for { Ale Gallons .00429627+ / Beer Bar. .000116741—

6. If the propoſed Veſſel be a Fruſtum of an *Octagonal Pyramid*; the ſaid reſerved Product divided by .6213203—, or multiplied by 1.609476+, either quotes, or produces the Solidity in Cubical Inches;

The proper Multiplicator for { Ale Gallons .00570737+ / Beer Bar. .0001585379+

7. If the Tunn propoſed, be a Fruſtum of a *Nonagonial Pyramid*, the ſaid reſerved Product divided by .4852935 —, or multiplied by 2.060609+, either quotes, or produces the Solidity in Cubical Inches; The

The proper Multiplicator for { Ale Gallons .0073071 3-+ / Beer Bar. .0002029756—

8. If the propoſed Veſſel be a Fruſtum of a *Decagonal Pyramid*, the ſaid reſerved Product divided by .3902434 —, or multiplied by 2.562503 —, either quotes, or produces the Solidity in Cubical Inches:

The proper Multiplicator for { Ale Gallons .00909481— / Beer Bar. .000252634-+

9. If the Tunn, inquired of, be a Fruſtum of an *Undecagonal Pyramid*, the ſaid reſerved Product divided by .3203199—, or multiplied by 3.121879—, either quotes, or produces the Solidity in Cubical Inches:

The proper Multiplicator for { Ale Gallons .01107048— / Beer Bar. .0003075138—

10. If the propoſed Veſſel be a Fruſtum of a *Dodecagonal Pyramid*, the ſaid reſerved Product divided by .2679492—, or multiplied by 3.732051-+, either quotes, or produceth the whole Tunns (or its Fruſtums) Solidity in Cubical Inches, according to the the tenour of the Queſtion:

The proper Multiplicator for { Ale Gallons .01323422— / Beer Bar. .000367617—

A General Rule for Inching any of the aforesaid Polygonial *and* Pyramidal Solids, *or* Mult-angled Tunns, *from the lesser Base to the greater; viz.*

1. To the tripled Side of the lesser Base add the Addend (found by the next foregoing Rule) and multiply their Summ by that Addend, then unto that Product add the tripled Square of the lesser Bases Side; (which done) that Summ multiplied by the given Multiplicator, proper to the Figure of the Tunn, and purport of the Question (whether the Content be sought in Gallons or Barrels) produces the Vessels first Frustums liquid Capacity, *viz.* at 1 Inches distance from the lesser Base.

2. To the nonocupled Side of the lesser Base add the septupled Addend; and their Summ multiply by the said Addend; then unto that Product add the tripled Square of the lesser Bases Side; (which done) that Summ multiplied by the said given Multiplicator, produces the first Difference.

3. To the sextupled Side of the lesser Base add the tripled Square of the Addend; (which done) that Summ multiplied by that Addend, and that Product, again, by the aforesaid Multiplicator, produces the second Difference.

4. The aforesaid Multiplicator multiplied by the sextupled Square of the Addend, produces the third Difference (which is identical throughout

out the Work) then proceed with the third Difference to make the ſecond and firſt Differences for every Inch of the Depth; and laſtly, the Table it ſelf.

EXAMPLE.

There is a Pyramidal Tunn, in the form of an Æquilateral Triangular Solid, *conſiſting of two unequal Baſes, but parallel, and alike ſituate; the three Sides at the top, being each* = 300 *Inches; at the bottom, each* = 180; *and the Depth,* = 30: *How many Beer Barrels may the whole Tunn contain, and how much upon each Inch of the Depth?* Fig. 3.

The Square of the Summ of the two Sides 300 and 180 = 230400

The Rectangle of the Sides = 54000

q. of Summ of Sides — their Rect. 176400

The Depth = 30

Product = 5292000

Multiplicator for Beer Barrels in an Æquilateral Triangulate = .0000142176 5—

5292000

2843530000
12795885
2843530
7108825

Reſp. Tunns whole Capac. 75.2398038 being 75 Bar. and 8 ½ Gall. *proximè.*

The Inching of the Tunn (by the given Rule) from the leſſer Baſe, to the greater.

The Side above = 300

The Side below = 180

Differ. of Sides = 120 (4 = the Addend.
Tunns Depth = 30

The tripled Side of the leſſer Baſe 180 = 540
The Addend = 4

Summ = 544
The Addend = 4

Product = 2176
The tripled q. of the leſſer Baſe 180 = 97200

Summ = 99376

Multiplicator for Beer Barrels in an Æquilateral Triangulate } = .00001421765

Product = 1.41289318 *&c.*
being the Tunns firſt Fruſtum.

Nonocupled Side of 180, the leſſer Side, = 1620
Septupled Addend, 4, = 28

Summ = 1648

Ad-

1648
Addend = 4

Product = 6592
The tripled q. of the leſſer Baſe, 180, = 97200

Summ = 103792
Multiplicator for Beer Bar. = .00001421765

Product = 1.47567832 *&c.*
The firſt Difference.

Sextupled Side of the leſſer Baſe, 180, = 1080
The tripled Square of the Addend, 4, = 48

Summ = 1128
Addend = 4

Product = 4512
The aforeſaid Multiplicator = .00001421765

The ſecond Difference = .06415003 *&c.*

The aforeſaid Multiplicator = .00001421765
Sextupled Square of 4, the Addend, = 96

8530590
12795885

The third Difference = .00136489440
of a Barrel.

Wet Inches.	Tunns Frustum from Inch to Inch in Beer Bar.	First Differences.	Second Differen.	Third Differen.
1	1.41289—			
		1.47568+	.06415—	
2	2.88857			.00136—
		1.53983	.06551	
3	4.42840			.00136
		1.60534	.06688	
4	6.03374			.00136
		1.67222	.06824	
5	7.70596			.00136
		1.74046	.06961	
6	9.44642			.00136
		1.81007	.07097	
7	11.25649			.00136
		1.88104	.07234	
8	13.13753			.00136
		1.95338	.07370	
9	15.09091			.00136
		2.02708	.07507	
10	17.11799			.00136
		2.10215	.07643	
11	19.22014			.00136
		2.17858	.07780	
12	21.39872			.00136
		2.25638	.07916	
13	23.65510			.00136
		2.33554	.08053	
14	25.99064			.00136
		2.41607	.08189	
15	28.40671			.00136
		2.49796	.08326	
16	30.90467			.00136
		2.58122	.08462	
17	33.48589			.00136
		2.66584	.08599	
18	36.15173			.00136
		2.75183	.08736	
19	38 90356			.00136
		2.83919	.08872	
20	41.74275			.00136
		2.92791	.09009	
21	44.67066			.00136
		3.01800	.09145	
22	47.68866			.00136
		3.10945	.09282	
23	50.79811			.00136
		3.20227	.09418	
24	54.00038			.00136
		3.29645	.09555	
25	57.29683			.00136
		3.39200	.09691	
26	60.68883			.00136
		3.48891	.09828	
27	64.17774			.00136
		3.58719	.09964	
28	67.76493			.00136
		3.68683	.10101	
29	71.45176			
		3.78784		
30	75.23960			

This, or any other Table may very easily be reduced into the form of the Practical Table, in *Pag.* 168. *Note*

Note also, That the first Differences answer the latter part of the Question.

There is a Pyramidal Tunn, in the form of a Pentagonal Solid, *consisting of two unequal Bases, but parallel, and alike situate; the 5 sides at the top, being each = 300 Inches; at the bottom, each = 180; and the Depth, = 30: How many Beer Barrels may the whole Tunn contain, and how much remains therein, when only 5 Inches of the Depth are dry?* Fig. 6.

The Square of the Summ of the Sides 300, and 180 = 230400

Rectangle of 300 and 180 = 54000

Remainder = 176400

The whole Depth = 30

Product = 5292000

Multiplicator for Beer Bar in a Pentagonal Solid — .0000564906+

1129812000
5084154
1129812
2824530

Resp. Tunns whole Capac. 298.9482552000 being 298 Beer Barrels, 3 Firkins, 7 Gallons, 1 Pint, *&c*

The

The Sides at top = 300

Sides at bottom = 180

Their Difference = 120 (4 the Addend or Subducend upon each Inch.

Depth = 30

4 the Addend.
5 the dry Inches.

Prod. 20 which taken

from 300 the Side at the top,

reſts 280 = the Side, at the Liquors ſuperficies.

The Sides of the Tunns propoſed Fruſtums are = 280, = 180

Their Summ = 460

460

27600
1840

Square of Summ of the Sides = 211600

Rectangle of the Sides = 50400

Remainder = 161200

Tunns wet Inches = 25

806000
322400

Product = 4030000

Multiplicat. for Beer Bar. .0000564906
4030000

1694718000
22596240

Resp. Remaining in Tunn = 227.657118
or 227 $\frac{1}{2}$ Beer Bar. and 5 $\frac{1}{2}$ Gall. *proximè.*

This, or any other of the *Polygonial Pyramidal Solids*, may be Inched according to the Rule last given: But this Discourse is here by way of Digression, and now we judge it requisite to return to *Cask Gauging*.

LXIII. Probl.

Probl. 1. *upon the Use of the Table, as in Pag.* 231.

Having a Casks liquid Capacity, and the Boung Diameter, together with its dry or wet Inches; To find the Vacuity, &c.

Analogism.

1. As the proposed Casks Boung Diameter, is unto its dry, or wet Inches, (according to the purport of the Question) so is 1.0000, (the Tabular Circles supposed Radius) unto a fourth Proportional; which found in the Table, in some Column under the Title, *Arrow*, the Table doth in the next Column on the right hand, shew the Area of the Tabular Circles Segment, cor-

correſpondent unto that of the Casks Boung Circle inquired of.

2. As 1.0000, the Tabular Semi-Circles Area; is unto the Tabular Area of its Segment, (afore found:) So is the whole liquid Capacity of the propoſed Cask; unto its Vacuity (in caſe the dry Inches were the ſecond term in the firſt Proportion;) otherwiſe, to the remaining Gallons in the Cask, the thing ſought.

For avoiding of Diviſions, the Boung Diameter (which is the Diviſor) may be ſuppoſed a vulgar Fraction, as the Boung Diameter propoſed in the firſt Queſtion, 25 Inches, may be ſuppoſed $\frac{1}{25}$; if Cyphers at pleaſure be poſtponed to the Numerator, and that Numerator (ſo increaſed) divided by its Denominator, as in the Margint, the Quotient .04 is the proper Multiplicator, by which the Product of the ſecond and third Terms in the firſt Proportion being multiplied, produces the fourth Proportional ſought; ſo for 29 is found .0344827+: Thus may in a ſhort time a ſufficient ſtore for Boung Diameters be laid up.

1 (.04
25

But in caſe the fourth place of the Tabular Circles Radius, to wit, under the Title (*Arrow*) happen to be any of the Digits, as 1, 2, 3, 4, 5, 6, 7, 8, or 9, and ſo not in the Table, as .2775, take the next leſs in the Table, *viz.* .2770, againſt

.2770 = .22578
Diff. .00114

againſt which (under the Title *Common Difference*) find .00114.

Then as the Difference .0010 to the Common Difference .00114; ſo is .0005 the exceſs of that fourth place, unto .00057.

.0010 : .00114 :: .0005 : .00057 which added to *per* T. = .2770 = .22578

And found .2775 = .22635

LXIV. Probl.

Probl. 2. *upon the Uſe of the Table, as in Pag.* 232.

Having a Casks liquid Capacity, and Boung Diameter, together, either with the Vacuity, or Quantity of Liquor, remaining in the Cask; To find the Boung Diameters dry or wet part.

The Analogiſm.

1. As the propoſed Casks Capacity, is unto its Vacuity, or, to the quantity of remaining Liquor; ſo is 1.0000 (the Tabular Semi-Circles Area) unto a fourth Proportional; which found in the Table, in ſome Column under the Title, *Area of Segment*, the Table doth in the next Column on the left hand ſhew the Arrow of the Tabular Circles Segment, correſpondent unto that of the Casks Boung Circle inquired of.

2. That Tabular Arrow (ſo found) being mul-

multiplied by the propounded Casks Boung Diameter (in Inches) produces always that Boung Diameters dry, or wet Inches, according to the purport of the Queſtion.

For avoiding of Diviſion, find for the propounded Casks liquid Capacity (according to the precedent Rule) the proper Multiplicator, and the ſame (ſo found) multiply by the Vacuity, or remaining Liquor propoſed, and that Product is equal to the fourth Proportional, above-mentioned; wherewith proceed according to direction above given; as by the ſubſequent Examples is made evident.

But in caſe the fourth Proportional, or Area of Segment produced, be not in the Table, then take that in the Table (which being leſs) comes nigheſt to it; obſerving their Difference, and noting the Arrow in the Table, correſpondent unto that leſſer Area, taken, as alſo the Tabular Common Difference of Area's againſt it, remembring that the Common Difference of the Arrows in the Table is always 10. Then

As the Common Difference of Area's (given in the Table) is to your obſerved Difference of Area's: ſo always 10 (the Common Difference of Arrows appearing by the Table) unto a fourth Proportional; the which being added unto the Arrow, correſpondent to the leſſer Area, taken; and that Summ multiplied by the propoſed Casks Boung Diameter, the Product ſo accruing, is always equal unto the Casks Boung Diameters dry or wet Inches; according

to

to the purport of the Queſtion, as by the following Examples will be manifeſt.

A Graves Hogſheads Boung Diameter being 25 Inches, and whole Capacity 63 Wine Gallons: How many Gallons, &c. are wanting, when 3 Inches of the Boung Diameter are dry, and the Axe poſited parallel to the Horizon? Probl. 63. Queſt. 1. *Fig.* 14.

Multiplicator for 25 = .04
Dry Inches = 3

Product = .12

.1200 = *per* T. = .06797—
The whole Capacity = 63

20391
40782

Reſponſ. 4.28211

Being 4 Gall. 1 Quart, and $\frac{1}{4}$ of a Pint, *quam proximè.*

If 58.71789 Wine Gallons do remain in a Cask, whereof the whole Capacity is 63 Wine Gallons, and Boung Diameter 25 Inches: How many of thoſe 25 Inches are wet? Probl. 64.

Multiplicator for 63 = .01587302
Wine Gallons = 58.71789

Product = .93203

.93203

.93203 = *per* T. .8800
The Casks Boung Diameter = 25

44000
17600

Respons. Wet Inches = 22(...

Probl. 63. Queſt. 2. *Fig.* 14. *A Wine Pipes Boung Diameter is* 29 *Inches, Head Diameter* 23 *Inches, and length* 48, *and whole Capacity (found by the Cyclometrical Table)* 120.2781 *Wine Gallons: Now if the* $\frac{1}{5}$ *of the Boung Diameter*, viz. 5.8 *Inches, be dry: How many Wine Gallons will fill up the Cask?*

120.278

Obſerving that the $\frac{1}{5}$ of the Tabular Circles Radius = .2000 I find againſt it *per* Table ——— } .14238

962224
360834
240556
481112
120278

Respons. 17.125 18164
being 17 Gall. and 1 Pint.

In-

Inversion.

If 17 *Gallons and* 1 *pint, poured into an empty Cask, whose Boung Diameter is* 29 *Inches, do make* $\frac{4}{5}$ *thereof, to wit,* 5. *Inches, wet: How many such Gallons will fill up this Cask, the whole Capacity being* 120.2781 *Gallons?* Probl. 63. Fig. 14.

Casks whole Capacity = 120.2780
$\frac{4}{5}$ of the Tab. Circles Radius being = .8000, I find against it *per* T. } .85762

240556
721668
841946
601390
962224

Respons. Wine Gall. = 103.15281836
Which with the Wants = 17.1252

Makes up the whole Capacity 120.2780 *&c.* as proposed.

There is an Oil Cask, whereof the Boung Diameter is 42 *Inches, Head Diameter* 31, *Length* 71 *Inches, its whole Capacity appearing by the Cyclometrical Table* 361.214 *Wine Gallons: Now* 16.8 *Inches of the Boung Diameter being dry, how many Wine Gallons are necessary to fill up the Cask?* Prob'. 63. Quest. 3. Fig. 14.

The Casks whole Capacity = 361.214
Casks dry Inches being $\frac{2}{5}$ of Boung Diam. I find *per* T. $\frac{2}{5}$ = .4000 = } .37353

1083642
1806070
1083642
2528498
1083642

Respons. 134.924 *&c.*
being 134 $\frac{1}{2}$ Wine Gall. 3 Pints and $\frac{1}{3}$ wanting.

Probl. 63. Quest. 4. *Fig.* 14. *There is a Rheniſh Wine Fatt, whereof the Boung Diameter is* 58 *Inches, Head Diameter* 46, *Length* 102 *Inches, and whole Capacity found by the Cyclometrical Table* 1022.3664 *Wine Gallons: Now if the dry Inches be* 10, *how many Wine Gallons are wanting?*

Multiplicator for the Boung Diam. = .01724
The dry Inches = 10

Product = .17240

.0010 : .00096 :: .0004 : .00039

I find *per* T. .1720 = 11467
To which I add the fourth Proportional = 39

And find = 1724 = 11506

The

Wine Gall.

The Casks whole Capacity = 1022.3664
Tabular Area = .11506

61341984
511183200
10223664
10223664

Resp. Wanting Wine Gall. 117.633477984
being 117 $\frac{1}{2}$ Gall. and 1 Pint.

The QUESTION inverted.

If 117.6335 *Wine Gallons be taken out of the Rhenish Wine Fatt (last mentioned:) How many are the dry Inches?* Probl. 64. Fig. 14.

Wine Gall.

The Ullage or Casks Vacuity = 117.6335
Multiplicat. for 1022.3664 = .000978123 +

Which being not in the Table .11506 *&c.*

The next less is .1720 *per* T. = .11467

Difference = .00039

Then

.00096 : .00039 :: .0010 : .0004

Which said fourth Proportional = .0004
Added unto the Arrow in the Table *viz.* .1720

Makes .11506 *per* T. = .1724

The Arrow = .1724
The Boung Diameter = 58

13792
8620

Resp. 9.9992
Or 10 Inches, *quam proximè, prout suprà.*

Probl. 63. Quest. 5. Fig. 14. *A Runlets whole Capacity being* 18 *Wine Gallons, and Boung Diameter* 17 *Inches: How much of the Liquor is wanting, when* 7 ½ *Inches of the Boung Diameter are dry?*

Mult. for Boung Diam. 17 Inch. = .0588235
The dry Inches = 7.5

2941175
4117645

Which being not in the Table *viz.* .44117625

The next less = .4410

excess.
.0010 : .00127 :: .000176 : .00022

.4410 = *per* Table .42505
Fourth Proportional = .00022

.441176 = .42527
The Casks whole Capacity = 18

Resp. Wanting Wine Gall. = 7.65486—
There

There is a Runlet, whose Boung Diameter is 17 Inches, Head Diameter 15, Length 19.779 Inches, and whole Capacity 18 Wine Gall Now if 10.34501 Wine Gallons be put into this Vessel, when empty; how many shall the Boung Diameters wet Inches be? Probl. 64. Fig. 14.

Mult. for whole Capac. 18 Gall. .0555556+
Liquor in the Vessel, Gall. = 10.34501

555556
27777800
2222224
1666668
5555560

Which not being in Table .574723237556

Next less = .558824 = .57368

104.33

.00126 : .0010433 :: .0010 : .000824

.57368 *per* Table = .5580
824

.558824

Boung Diameter = 17

3911768
558824

Resp. The wet Inches are = 9.500008

A Chymiſt having prepared a rich Elixir or Reſtorative Liquor, worth 10 l. per *Pint, reſerved it in a Runlet of the ſame Dimenſions, with that laſt mentioned: And finding (when the Veſſels Axe was poſited parallel to the Horizon) the Boung Diameters wet Inches* 15 $\frac{1}{2}$; *he delivers the ſame into his Servants cuſtody, who privily ſold a quantity of it, which the Chymiſt hears (though certainly knows not) was 7 Gallons, Wine Meaſure: He having not put the Liquor by meaſure into the Veſſel, deſires (if ſo much Liquor be imbezilled) to know how many the wet Inches are; As alſo, in caſe the wet Inches (when the Servant deliver'd up the ſame) were but* 9 $\frac{1}{2}$, *what the imbezill'd Liquor amounts unto at* 10 l. per *Pint, taking the Veſſel, as Spheroidal; and what the ſame comes to, if it be aſſumed, as the middle Fruſtum of a Parabolical Spindle.*

Mult. for Boung Diam. 17 Inches .0588235
The wet Inches = 15.5

2941175
2941175
588235

.91176425

com. diff. exceſs.
.0019 : .00072 :: .000764 : .00055

The

The Arrow .9110 *per* T. = .95615
Fourth Proportional = .00055

.91176425 = .9567
The Casks whole Capacity = 18

76536
9567

Delivered to the Servant Wine Gall. = 17.2206

Imbezill'd (as ſuppoſed) Gall. = 7

Then there had remained in the Veſſ. 10.2206 Wine Gallons.

Mult. for whole Capacity 18 Gall. .0555556
10.2206

3333336
11111120
1111112
5555560

.567811156536

.56735
46

.00126 : .0004616 :: .0010 : .00036

.56735 *per* T. = .5530
Fourth Proportional = .00036

.567811565536 = .55336
The Boung Diameter = 17

387352
55336

Resp. If the imbezill'd Liquor were 7 Gallons, the wet Inches are } 9.40712

Mult. for Boung Diam. 17 Inches .0588235
The wet Inches = 9.5

2941175
5294115

.55882325

.0010 : .00126 :: .000823 : .00104

.5580 *per* T. = .57369
Fourth Proportional = .00104

.558823 = .57473
The Veſſels whole Capacity = 18

459784
57473

When the wet Inches are 9 ½ there remains in the Veſſ. Wine Gall. } 10.34514

Which

10.34514

Which ſubducted from that which was deliver'd to the Servant, *viz.* } 17.2206

The imbezill'd Liquor (taking the Cask as Spheroidal) in caſe the wet Inches were $9\frac{1}{2}$, is } 6.87546
being 6 Gallons, and 7 Pints, *quam proximè*; amounting to 550 *l.* 00 *s.* 9 *d.*

It appears by Rule given in *Pag.* 181. that this Veſſel (if aſſumed as the middle Fruſtum of a Parabolical Spindle) contains Wine Gall. but 17.641513, the which being divided by 18 Gallons (its Capacity, as Spheroidal) quotes your Multiplicator, *viz.*

.980085
6.87546

3437730
5500368
550036800
6187914

Taking the Veſſel as Parabolical, and wet Inch $9\frac{1}{2}$ the imbezill'd Liquor is but } 6.738535214 10
being 5 Gallons, 5 Pints, $\frac{1}{4}$ and half, *proximè*; amounting to 539 *l.* 1 *s.* 8 *d. ferè.*

LXV.

LXV. Probl.

Having a Cask's Boung Diameter, and whole Capacity in Gallons, together with the Vacuity, or the quantity of remaining Liquor in it (as taken for the middle Fruſtum of a Spheroid;) To find either of them, in caſe the Cask be taken, as the middle Fruſtum of a Parabolical Spindle.

The Rule.

1. By direction of *Pag.* 267. find the quantity of remaining Liquor, or the Vacuity of the Cask, as Spheroidal.

2. By direction of *Pag.* 181. find the propoſed Casks whole Capacity, as the middle Fruſtum of a Parabolical Spindle; Then, As the given Capacity of the propoſed Cask (as Spheroidal) is to its correſpondent Fruſtum, alſo given: So is its Capacity found (as Parabolical) to its correſpondent Fruſtum required.

Or, rather

Divide the propoſed Casks liquid Capacity (as Parabolical) by its Capacity, as Spheroidal; which may be performed by multiplication; when the Multiplicator is found by Rule in *Pag.* 268. and that Quotient or Product, multiplied by any Fruſtum of this propoſed Cask (as Spheroidal) reduces it into its correſpondent Fruſtum, as Parabolical, the thing required.

Queſt.

Queſt. I.

There is a Graves Hogſhead, whereof the Boung Diameter is 25 *Inches, and whole Capacity* 63 *Wine Gallons; which being aſſumed for the middle Fruſtum of a Spheroid, lying with its Axe parallel to the Horizon; is found (when* 3 *Inches of the Boung Diameter are dry) to have remaining in it of Wine Gallons* 58.7177: *The Queſtion is, how many ſuch Gallons remain in this Cask, at the ſame poſition and altitude of Liquor, in caſe the Cask be taken, as the middle Fruſtum of a Parabolical Spindle.*

Proper Multiplicat. for this Cask .9792388+
Remaining Liquor = 58.7177

68546716
68546716
9792388
68546716
78339104
48961940

Reſponſ. Wine Gallons 57.49865008676;
The whole Cask containing 61.69206—

Queſt. II.

There is a Wine Cask, whereof the Boung Diameter is 67 $\frac{1}{2}$ *Inches, and whole Capacity*

1352.091 *Wine Gallons, which being assumed for the middle Frustum of a Spheroid, is found (when* 40½ *Inches of the Boung Diameter are wet) to have remaining in it, of Wine Gallons* 505.0471—: *The Question is, how many such Gallons remain in this Cask, at the same altitude of Liquor, if the same be taken as the middle Frustum of a Parabolical Spindle?*

Multiplicator for this Cask = .8667723+
Remaining Liquor = 505.0471—

8667723
60674061
34670892
4333861 50
4333861 50

Resp. Wine Gallons = 437.76083647533

The whole Cask containing 1171.9546— Wine Gallons.

LXVI. Probl.

Having a Casks Boung Diameter and whole Capacity in Gallons, together with the Vacuity or quantity of remaining Liquor in it (as taken for the middle Frustum of a Spheroid;) To find either of them, in case the Cask be taken, as the middle

middle Fruſtum of two Cones, abutting upon one common Baſe, &c.

The Rule.

According to Direction in *Pag.* 183. find the propoſed Casks whole Capacity, as the middle Fruſtum of two Cones, *&c.* then divide the propoſed Casks liquid Capacity (as Conical) by its Capacity (as Spheroidal;) which done, that Quotient being multiplied by any Fruſtum of the propoſed Cask (as Spheroidal) reduces it unto its correſpondent Fruſtum (as Conical) the thing required.

As for Example.

There is a Graves Hogſhead, whereof the Boung Diameter is 25 *Inches, and whole Capacity* 63 *Wine Gallons; which being aſſumed for the middle Fruſtum of a Spheroid, lying with its Axe parallel to the Horizon, is found (when* 3 *Inches of the Boung Diameter are dry) to have remaining in it Wine Gallons* 58.7177: *The Queſtion is, how many ſuch Gallons remain in this Cask at the ſame poſition and altitude of Liquor, in caſe it be taken, as the middle Fruſtum of two Cones, abutting upon one common Baſe,* &c.

Mul-

Multiplicator for this Cask, is = .9567478 +
58.7177

66972346
66972346
9567478
66972346
76539824
47837390

And .9218119 for a Canary ipe.

Resp. Wine Gallons = 56.17803029606

Casks Capacity (as Conical) being 60.27510 Wine Gallons.

LXVII. Probl.

Quest. 1. *A Cylindrical Casks Diameter is* 25 *Inches, Length* 32, *and whole Capacity* 68 *Wine Gallons: Now if when this Cask lye with the Axe parallel to the Horizon,* 3 *Inches of the Diameter be dry; how many of the* 68 *Gallons are wanting?*

Multiplicator for Diameter 25 = .04
Dry Inches = 3

.12

I find .1200 *per* Table = Area .06797
The Casks whole Capacity = 68

54376
40782

Resp. Wanting, Wine Gallons = 4.62196

LXVIII. Probl.

There is a Cylindrical Cask, whereof the whole Capacity is 68 Wine Gallons, the Diameter of its Base 25 Inches, which lying with the Axe parallel to the Horizon, there remains in the Cask Wine Gallons 63.37804 : how many of the 25 Inches Diameter, are wet? Queſt. 1.

Remaining Liquor = 63.37804
Multiplicator for whole Capacity .0147059+

57040236
31689020
443646280
253512160
6337804

.932033118436
.93203 *per* T. = Arrow .8800
Diameter of the Base = 25

44000
17600

Reſponſ. Wet Inches = 22

LXIX.

Probl. 67. Queſt. 2

Fig. 13. *There is a Cylindrical Cask (which let be repreſented by the* Figure, d, d, w, u,) *whereof the Baſes Diameter,* D, m, *being* 23 ½ *Inches, and* i, o, *the Length,* = 32, *the whole Capacity is* 60.0864 *Wine Gallons: Now in caſe this Veſſel lie with the Axe parallel to the Horizon, and* D, y, (viz. *the dry part of the Baſes Diameter*) *be* = 4.7 *Inches; what is the Ullage, or how many of the* 60 &c. *Gallons are wanting?*

Multiplicator for Diam. 23 ½ = .0425532+
D, y, = 4.7

2978724
1702128

Arrow = .2000(0004

To the Arrow .2000, the proper Area of Segment *per* Tab. = .14238
The Casks whole Capacity = 60.0864

56952
85428
113904
8542800

Reſp. 8.555101632
being 8 ½ Gallons, and nigh half a Pint.

Probl. 68. Queſt. 2.

Of the Cylindrical Cask, laſt mentioned, lying with the Axe parallel to the Horizon, the whole Diameter of the Baſe, D, m, *being* 23 $\frac{1}{2}$ *Inches, whole Capacity* = 60.0864 *Wine Gallons, and the Ullage, or Wants* = 8.5551 — *Wine Gallons: What is the Quantity of* D, y, *or how many are the dry Inches?* Fig. 13.

The Multiplicator for the whole Capacity 60.0864 } = .0166427—
The Ullage given = 8.5551—

166427
832135
832135
832135
1331416

Area of Segment = .1423799 *&c.*

To the Area of the Segment .14238 the proper Arrow *per* Table } = .2000
The Casks Diameter propoſed = 23.5

Reſp. Dry Inches = 4.70000

Such as are expert in Diviſion may work the Solution of this, or other *Problems*, thus, *viz.*

480691
228328
143005
2 54646
60.0864)8.555101632(.14238 Area of
6 00864 Segment.
2 403456
1201728
1802592
4806912

Proof 8.555101632

Or thus:

60.0864) 8.555101632 (.14238
2 546461 60.0864
1430056
4806912 8.555101632
.....

Then (having found the Area of the Segment) finish the Work according to the precedent Examples.

The

The Uſe of the Inſtrument.

THe Inſtrumental Operation of any *Problem* in this Book, (but more eſpecially intended for thoſe relating to the Gauging of a Cask partly full, lying with the Axe parallel to the Horizon, the Superficies of the Liquor cutting the Heads) may be performed after a facile, expeditious, and moſt practical way, by aid of a ſliding Ruler, projected by this Author, but contrived, made and ſold by *John Brown*, Mathematical Inſtrument-Maker, at the *Sphear and Sun-Dial* in the *Minories, London:* Of which Ruler you have here a Figure repreſented.

A Brief Deſcription, together with ſome few of this Inſtruments manifold Uſes, with apt, uſeful, and practical Inſtances thereupon, viz.

ON the firſt Edge are two Lines, the firſt graduated from 1 Inch to 18, in Inches and Decimal Parts; and the other Line, of 7 Inches, divided into 100 equal Parts, for finding the vulgar equated Diameter of an ordinary cloſe Cask.

On the other, or second Edge, are also two Lines; the one, graduated from 1 to 18 Inches; and the other, backwards, from 18 to 36 Inches, and Decimal Parts; of use in taking the Dimensions either of any superficial Figure, or Solid Body, proposed; and to be used either singly or as a sliding Ruler.

1. On the right side of the Ruler (against *Pence*) is a Line, shewing the Decimal Parts of 1 *l Sterling*, of great use for the speedy computation of the value of any number or assigned quantity of Goods or Merchandize at any rate proposed.

2. & 3. Against *Numb.* abutting on each side of the sliding Joint, is placed the single Line of Numbers, upon each twice repeated, numbred from 1 at the beginning to 1 or 10 in the middle; and then with 2, 3, 4, *&c.* to 10 or 100, at the end; of use in Multiplication, Division, *&c.*

4. Against *Segment* is placed a Line of artificial Segments of a Circle; of use in finding of the Area of any Circles Segment inquired of, either in Square Inches, or in Wine or Ale Gallons.

5. And under that against *Segment*, (is added by the Instrument-maker) a Line of Segments of a Sphear or Globe; of use in finding the solidity of a Globes or Sphears Segment. *Vide Brownes Triangular Quadrant, Pag.* 169, 170; and 251, 252, 253.

On

On the Back, or other ſide of the Ruler.

Againſt *Doub. N.* is placed a double Line of Numbers, or a Line of Numbers twice repeated: And on the other ſide of the ſliding Joint, againſt *Broken*, is the broken Line of Numbers; ſerving together for the Extraction of the Square Root (as by their Inſcription at the other end of the Ruler appears) and alſo for the Solution of ſuch Queſtions, as require a double Operation. 1. & 2.

Againſt *Single*, is a ſingle Line of Numbers; and next to it, againſt *Triple*, is a Line of Numbers thrice repeated, ſerving together for the Extraction of the Cubick Root, (as is hinted at the Rulers other end.) 3. & 4.

Two neceſſary Notes to be obſerved in uſing of this Sliding RULER.

1. Note, That in uſing the Line of Numbers, that Line or Radius of it, on which you find the firſt term in the Queſtion, is always to be accounted the firſt Line, or Radius; and the other conſequently, the ſecond.

2. To find the Point upon the ſingle Line of Numbers, repreſenting any Number ſought; obſerve, you muſt ſeek the firſt Figure of the Number propoſed, among the long ſtrokes with Figures annexed, and the ſecond Figure of the Number, among the longeſt ſtrokes next between

between that and the next Figure upon the Rule and the third Figure of the Number, among the ſhorteſt ſtrokes, between the two Lines on the Ruler, and between the Figure, or place laſt found and the next ſubſequent Figure on the Ruler; but for the fourth or laſt Figure (if ſo many there be) it muſt be counted only by computation, on this, and all other Rulers of ſo ſhort a Radius as 12 Inches.

As for INSTANCE.

To find the Point on the ſingle Line of Numbers, repreſenting the Number 125.

The 1 appears at the beginning of the Radius; then 20 is two tenths further, and for the 5, count ſo many ſhort cuts further, being directly over 2 in the triple Line: But this Number is found on the triple Line, directly againſt 5 on the ſingle Line of Numbers.

To find on the ſingle Line of Numbers, the Point repreſenting the Number 2715.

Reckon the firſt Figure 2, at 2 on the Line, and 7, at ſo many tenths, between 2 and 3, and the 1, at 1 ſhort Cut or Diviſion beyond that, and the laſt, 5, at juſt half way between the firſt and ſecond Diviſion: So have you the Point of 2715 on the Line of Numbers, being directly againſt 20, on the triple Line; and further, on ſo ſhort a Ruler, (*viz.* as 12 Inches) can be no exact computation. Here

Here likewiſe obſerve, That the parts between 3 and 6, are two, and between 6 and 10, only every half part, as the Line may bear; yet thoſe places are to be eſtimated, as cut into as many parts as between 1 and 2.

The Uſe of the Sliding Ruler, which I ſhall here hint at, is ſevenfold, *viz.*

The Uſe of this Ruler in

Multiplication,
Diviſion,
The Rule of Proportion Direct,
Extraction of the Square Root,
The finding of Geometrical Mean Proportionals,
Extraction of the Cubick Root,
Gauging of a Casks vacant or replete Fruſtum, lying with the Axe parallel to the Horizon.

And of each in their order.

I. *The Uſe of this Ruler in Multiplication.*

As Unity, or 1, is to the Multiplicator; ſo is the Multiplicand, to the Product.

Draw the ſliding Ruler, till 1 on any ſide of the Line of Numbers (which call the firſt Side or Line) do cut or interſect the Multiplicator on the other, or ſecond Side, or Line; (which done) the Multiplicand being found on the firſt Line, ſhews (directly againſt it on the ſecond Line) the Product ſought.

Example.

To multiply 145 *by* 8.

Draw the Ruler, till 1 on one ſide of the Line be juſt againſt the Multiplicator 8 on the other, then right againſt the Multiplicand (here 145) found on the firſt ſide appears 1160 on the ſecond Line or Side, being the Product required. Thus 9 multiplied by 8, produces 72; and 455, by 97, produces 44135: But if you uſe this Printed Figure with a pair of Compaſſes, then extend the Compaſſes from 1 to the Multiplicator; (which done) ſetting one Foot of the Compaſſes upon the Multiplicand, keeping the ſame extenſion, the other Foot ſhall reach exactly unto the Product.

But for knowing the certain number of places in the Product (upon ſight only of the Multiplicand and Multiplicator;) and for aſcertaining the two laſt of four or five (and ſometimes of ſix places;) *vide* Brownes *Triangular Quadrant*, *Pag.* 84, and 85.

II. *The Uſe of this Ruler in Diviſion.*

Set the Diviſor on the firſt ſide, to 1 on the ſecond; then directly againſt the Dividend (to be found on the firſt ſide or Line) appears on the ſecond Line the Quotient ſought: Thus 42 divided by 7, quotes 6.

Example.

Example

What is the Quotient of 135 *divided by* 9?

Set the Diviſor 9 on the firſt ſide, unto 1 on the ſecond; then juſt againſt 135, the Dividend, on the firſt ſide, ſtands 15 on the ſecond ſide; being the Quotient required.

But if you uſe this Printed Figure, then extend the Compaſſes from the Diviſor, unto 1; (which done) the ſame extent of the Compaſſes, applied (always) the ſame way from the Dividend, ſhall exactly reach unto the Quotient required.

Note here, That in working with the Compaſſes, you may uſe the ſingle and broken Line of Numbers, either croſs-wiſe, from one Line to another, or work right on upon any one of them.

And for knowing the certain number of places in any Quotient (upon ſight only of the Dividend and Diviſor) *vide* Brownes *Triangular Quadrant*, *Pag.* 97.

III. *This Rulers uſe in working the Rule of Proportion Direct.*

Seek the firſt Term of the Queſtion, on any ſide of the Line of Numbers (which call the firſt ſide) and draw the ſliding Ruler, till the firſt Term, on the firſt ſide, interſect with the ſecond Term on the ſecond ſide; (which done) directly

directly againſt the third Term (to be found out on the firſt ſide) ſtands on the ſecond ſide the fourth Proportional, which anſwers the Queſtion.

As for Example.

If 375 Yards of Scarlet coſt 2000 l. what coſt 1 Yard?

Set 375 on one ſide, to 2000 on the other; then directly againſt 1 on the firſt ſide, ſtands on the ſecond ſide 5333 *&c.* being 5.333, which is according to the Doctrine of Decimals 5 *l.* 6 *s.* 8 *d.* for the price of the Yard of Scarlet, ſought.

IV. *The uſe of this Ruler in the Extraction of the Square Root.*

Point the Square Number, putting a Point under the laſt place to the right hand, and the third next, the fifth next, the ſeventh next, the ninth next, if ſo many there are.

Every Square Number muſt conſiſt either of an odd or an even number of places: Where note, That if the Square Number propoſed, do conſiſt of an odd number of places, as 1, 3, 5, 7, or 9 places; that Square Number being found in the ſecond Radius, or part of the double Line of Numbers, (there doth directly againſt the ſame) on the broken Line of Numbers, appear the Square Root, ſought. Thus the

Square

Square Root of 9, appears 3; of 144, 12; of 12544, 112; and of 96845544, 3112.

But if the propounded Square Number consist of any even number of places; as 2, 4, 6, or 8 places; that Square Number being found in the first part, or first Radius of the double Line of Numbers, there doth (just against it) on the broken Line of Numbers, stand the Square Root required: Thus the Square Root of 81, appears 9; of 9025, 95; of 970225, 985; *&c.*

V. *Between any two Numbers given, To find a Geometrical Mean Proportional, by aid of this Ruler.*

Take the distance between the two Numbers given, (counted on the double Line of Numbers,) and set the same from one of these Numbers towards the other, (on the broken or single Line,) and it shall shew the Geometrical Mean Proportional Number sought.

As for Instance.

Let 8 *and* 48 *be proposed, and a Geometrical Mean Proportional between them, be required.*

Upon the double Line of Numbers extend the Compasses from 8 to 48, (which done) that extension shall (upon the broken or single Line

Line of Numbers) reach from 8 unto 19.6 *ferè*, which is the Geometrical Mean, ſought.

VI. *Between two Numbers given; To find two Geometrical Mean Proportionals, by the Ruler.*

Upon the triple Line of Numbers take the diſtance between the two given Numbers; then on the broken Line of Numbers ſet the ſame extenſion from one of thoſe Numbers, towards the other, and again turn the Compaſſes, noting where the two Points at each turning do ſtay, for the two Geometrical Means, ſought.

As for Example.

Let 4 and 32 be the two Extream Numbers propoſed; and two Geometrical Mean Proportionals between them be ſought.

Take on the Triple Line of Numbers the diſtance between 4 and 32: (which done) the ſame extenſion applied (on the broken Line) from 4 towards 32, doth at one turning reach to 8, and at the ſecond turning unto 16; which are the two Mean Proportionals, ſought.

For further Inſtruction, *vide* Brownes *Triangular Quadrant*, *Pag.* 104. and 116.

VII. *Between two Numbers given; To find (by this ſliding Ruler) 3, 4, or 5 Geometrical Mean Proportionals, or as many as you pleaſe.*

Admitting the Numbers be as 1 to 2; draw the Ruler, till 1 on any one ſide of the Line of Num-

Numbers ſtands directly againſt 2 on the other ſide; (which done) juſt againſt 2, on the firſt ſide, (where 1 was taken) ſhall be 4 on the ſecond ſide; and directly againſt 4 on the firſt ſide, ſhall be 8 on the ſecond; and juſt againſt 8 on the firſt ſide, is 16 on the ſecond; and ſo as far as you will: So 4, 8, 16, 32, 64, are the five Geometrical Means, ſought.

VIII. *The Uſe of this Sliding Ruler in the Extraction of the Cubick Root.*

1. Point the Cube Number propoſed, putting a Point under the laſt place to the right hand, (which reckon here as the firſt) another under the fourth, another under the ſeventh, and another Point under the tenth place (if ſo many there be) whence the number of places whereof the Cubick Root ſought conſiſts, doth appear.

2. Note, That if the Cube Number propoſed, do conſiſt of 1, 4, or 7 places, then ſeek it on the firſt Radius or part of the triple Line of Numbers; if of 2, 5, or 8 places, on the ſecond Radius; and if of 3, 6, or 9 places, the Cube Number muſt be ſought on the third Radius of the triple Line of Numbers; (which done) in all the Caſes there doth directly againſt each Cube Number (ſo found as aforeſaid) ſtand on the ſingle Line of Numbers, the reſpective Cubick Roots required. Thus the Cubick Root of 8, appears 2; of 1728, 12; of 27, 3; of 27000, 30; *&c.*

IX.

IX. *Cask-Ganging by the Sliding Ruler.*

And first of a whole Cask, *viz.*

Queſt. I.

Fig. 13. *A Graves Hogſheads Boung Diameter is* 25 *Inches, Head Diameter* = 22, *and Length* = 32.058+ : *How many Wine Gallons may this whole Cask contain?*

Firſt ſet 1.0 *&c.* on the Line of Numbers, by that of a Circles Segments, unto .7 on the other ſide of the Line of Numbers; (which done) directly againſt the Difference of the Boung and Head Diameters (being here 3 Inches) ſtands on the firſt ſide or Line, a fourth number, (here 2.1) which added unto the Head Diameter, (here 22) makes a number, which call the equated Diameter, (being here 24.1).

Then turn the Ruler.

The Capacity being ſought in Wine Gallons, ſet *w* (being 17.15 the Gauge-point for a Wine Gallon) on the broken Line of Numbers, exactly againſt the Veſſels length (here 32.058) on the double Line of Numbers; (which done) directly againſt the equated Diameter (here 24.1) on the broken Line of Numbers, ſtands on the double Line of Numbers, the Casks whole Capacity in Wine Gallons, the thing ſought, (here being 63.2;) which is too much only by $\frac{2}{10}$ of a Gallon.

Here note, That if the Capacity had been ſought

sought in Ale Measure, the Gauge Point A must have been used.

Quest. II.

A Graves Hogsheads Boung Diameter, D, m, *being* 25 *Inches, and whole Capacity* 63 *Wine Gallons: What is the Ullage, or how many Gallons*, &c. *are wanting, when the dry part of the Boung Diameter*, viz. D, r, *is* 3 *Inches, the superficies of the Liquor*, V, r, q, *cutting the Heads* d, u, *and* d, w, *and the Casks Axe*, i, C, o, *being posited parallel to the Horizon?* Fig. 14.

1. Seek the proposed Casks Boung Diameter, upon the Line of Numbers (by that of a Circles Segments) and draw the Sliding Ruler, until it cut 1.00 upon the Line of a Circles Segments, on the other side; which done, seek the proposed dry or wet Inches on the same Line, where the Boung Diameter was taken, and against it on the Line of Segments is the Area of the Tabular Segment, (here .068+) which reserve.

2. Upon either of the Lines of Numbers, set 1.00 *&c.* to the reserved Area (here .068) on the other side of the Line of Numbers; which done, seek the proposed Casks whole Capacity (here 63 Gallons) on the same line or side, where 1.00 *&c.* was set, and directly opposite thereto, is on the other side, either the Ullage, or the remaining Liquor sought, (here the Ullage 4.28) which according to the purport of the Question is 4.28 Wine Gallons.

Quest.

Quest. III.

Fig. 14. *There is a Cask (in part empty) lying with the Axe parallel to the Horizon, the superficies of the remaining Liquor*, V, r, q, *cutting the Heads*, d, u, *and* d, w, *the Vessels whole Capacity being 63 Wine Gallons, whereof it is known, that the Ullage, or Wants*, viz. *the vacant Frustums Capacity* V, d, D, d, q, r, *is* 4.28 *Gallons, and* D, m, *the Boung Diameter* 25 *Inches: What is the Quantity of* D, r, *or how many Inches of the Boung Diameter are dry?*

1. Upon the Line of Numbers set the Vessels whole Capacity (here 63 Gallons) directly against the Ullage (here 4.28 Gallons) on the other Line of Numbers; which done, directly against 1.0 *&c.* on the first side stands the Area of the Tabular Segment, which reserve, (being here .068+).

2. Set 1.00 *&c.* on the Line of a Circles Segments to the proposed Vessels Boung Diameter (here 25) on the Line of Numbers; which being done, directly against the said reserved Tabular Area .068 (found on the said Line of Segments) stands on the Line of Numbers, the Boung Diameters dry or wet part in Inches required (according to the purport of the Question,) being here 3 Inches, for the Quantity of *D, r*, the Boung Diameters dry part, sought.

And on this manner *(mutatis mutandis)* may any *Problem* in the Book have its Instrumental Solution.

FINIS.